LAST NAME: SOLUTIONS

FIRST NAME: ____________

STUDENT NUMBER: ____________

TEST 1 (A)

DAWSON COLLEGE

201-NYC-05 - Linear Algebra

Instructor: E. Richer

Date: June 19th 2008

Question 1. *(10 marks)*

(a) Define briefly the three types of elementary row operations.

(1) INTERCHANGING 2 ROWS

(2) MULTIPLYING A ROW BY A NON ZERO CONSTANT

(3) ADDING A MULTIPLE OF A ROW TO ANOTHER

(b) Using elementary row operations, find the **reduced row echelon** form of the following matrix. Show each step, writing a new matrix showing the result of each of your elementary row operations.

$$A = \begin{bmatrix} 0 & 1 & 1 & 4 \\ 1 & 1 & 2 & -1 \\ 3 & 4 & 7 & 1 \end{bmatrix} \quad R_1 \leftrightarrow R_2 \quad \begin{bmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & 1 & 4 \\ 3 & 4 & 7 & 1 \end{bmatrix}$$

$$R_3 - 3R_1 \to R_3 \quad \begin{bmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & 1 & 4 \\ 0 & 1 & 1 & 4 \end{bmatrix} \quad R_3 - R_2 \to R_3 \quad \begin{bmatrix} 1 & 1 & 2 & -1 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1 - R_2 \to R_1 \quad \begin{bmatrix} 1 & 0 & 1 & -5 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Question 2. *(10 marks)*
Consider the system whose augmented matrix is given in row echelon form below, where a is a constant:

$$\begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & a^2-9 & a-3 \end{bmatrix}$$

State the conditions on a and b so that the corresponding linear system has:
(i) No solutions
(ii) Infinitely many solutions
(iii) A unique solution

(i) $a^2-9=0$ & $a-3 \neq 0$

$a=\pm 3$ & $a \neq 3$

THIS LEAVES ONE POSSIBILITY

$\boxed{a=-3}$

(ii) $a^2-9=0$ & $a-3=0$

$a=\pm 3$ & $a=3$

THIS LEAVES only $\boxed{a=3}$

(iii) $a^2-9 \neq 0$

$a^2 \neq 9$

$\boxed{a \neq \pm 3}$

Question 3. *(10 marks)*
Solve the following system of equations.

$x_1 + 2x_2 - x_3 = 1$
$2x_1 + 4x_2 - 2x_4 = 6$
$-2x_1 - 4x_2 + 2x_3 = -2$

$$\begin{bmatrix} 1 & 2 & -1 & 0 & 1 \\ 2 & 4 & 0 & -2 & 6 \\ -2 & -4 & 2 & 0 & -2 \end{bmatrix} \quad \begin{matrix} R_2 - 2R_1 \to R_2 \\ R_3 + 2R_1 \to R_3 \end{matrix} \quad \begin{bmatrix} 1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 2 & -2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tfrac{1}{2} R_2 \to R_2 \quad \begin{bmatrix} 1 & 2 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

FREE VARIABLES ARE X_2 & X_4

Let $X_2 = s$
$X_4 = t$

$$X_3 - X_4 = 2$$
$$X_3 = 2 + X_4$$
$$= 2 + t$$

$$x_1 + 2X_2 - X_3 = 1$$
$$X_1 = 1 - 2X_2 + X_3$$
$$= 1 - 2s + (2+t)$$
$$= 3 - 2s + t$$

$$(X_1, X_2, X_3, X_4) = (3-2s+t,\ s,\ 2+t,\ t)$$

WHERE s, t ARE in $\mathbb{R}$.

Question 4. *(10 marks)*

$$A = \begin{bmatrix} 2 & -4 \\ 3 & 1 \\ -1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}$$

Compute the following (where possible).

(a) CA

(b) $A + B^T$

(c) $CB - 2A^T$

(a) $\underset{2\times 2}{C} \qquad \underset{3\times 2}{A}$ $\qquad$ CA is not defined

(b) $A + B^T$

$$\begin{bmatrix} 2 & -4 \\ 3 & 1 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 4 & 1 \\ -4 & -2 \end{bmatrix}$$

(c) $CB - 2A^T$

$$= \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & -2 \end{bmatrix} - 2\begin{bmatrix} 2 & 3 & -1 \\ -4 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 & 1 \\ 2 & 3 & -13 \end{bmatrix} - \begin{bmatrix} 4 & 6 & -2 \\ -8 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -5 & 3 \\ 10 & 1 & -13 \end{bmatrix}$$

Question 5. *(10marks)*
Find A given the information below.
$$(A-2I)^{-1} = \begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}$$

$$\left((A-2I)^{-1}\right)^{-1} = \begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix}^{-1}$$

$$A-2I = \frac{1}{10}\begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2/10 & 1/10 \\ -4/10 & 3/10 \end{bmatrix} + 2I$$

$$A = \begin{bmatrix} 2/10 & 1/10 \\ -4/10 & 3/10 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 22/10 & 1/10 \\ -4/10 & 23/10 \end{bmatrix}$$

Question 6. *(10 marks)*
Given n x n matrices A and B. State whether or not the following statements are TRUE or FALSE. If the statement is false give an example otherwise simply state that the statement is true.

(a) $(A+B)^{-1} = A^{-1} + B^{-1}$

FALSE

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(A+B)^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

BUT $A^{-1} + B^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

(b) $(AB)^{-1} = B^{-1}A^{-1}$

TRUE

(c) $(AB)^T = B^T A^T$

TRUE

(d) $AB = BA$ FALSE

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$AB = \begin{bmatrix} 0 & 3 \end{bmatrix}$ $BA = \begin{bmatrix} -1 & 1 \end{bmatrix}$

Question 7. *(10 marks)*
(a) Given a square matrix A, state the two equations that a matrix C must satisfy in order for $C = A^{-1}$.
(b) Show that if a square matrix satifies $A^2 + 2A - I = 0$, then $A^{-1} = A + 2I$

(a) (1) $CA = I$
(2) $AC = I$

(b) We must show $A^{-1} = A + 2I$
so we must show
(1) $(A+2I)A = I$
& (2) $A(A+2I) = I$

We are given $A^2 + 2A - I = 0$
that is $\boxed{A^2 + 2A = I}$

Checking (1) $(A+2I)A$
$= A^2 + 2IA$
$= A^2 + 2A$
$= I$ (FROM given)

(2) $A(A+2I)$
$= A^2 + A2I$
$= A^2 + 2A = I$

So $(A+2I)$ satisfies both conditions for being the inverse of A
We can conclude that $A^{-1} = A + 2I$