LAST NAME: SOLUTIONS

FIRST NAME: ____________

STUDENT NUMBER: ____________

TEST 1 (B)

DAWSON COLLEGE

201-NYC-05 - Linear Algebra

Instructor: E. Richer

Date: June 19th 2008

Question 1. *(10 marks)*

(a) Define briefly the three types of elementary row operations.

(1) INTERCHANGE 2 ROWS

(2) MULTIPLY A ROW by A NON-ZERO CONSTANT

(3) Add A Multiple of one row to Another

(b) Using elementary row operations, find the **reduced row echelon** form of the following matrix. Show each step, writing a new matrix showing the result of each of your elementary row operations.

$$A = \begin{bmatrix} 0 & 2 & 2 & 8 \\ 2 & 2 & 4 & -2 \\ 6 & 8 & 14 & 2 \end{bmatrix} \quad R_1 \leftrightarrow R_2 \quad \begin{bmatrix} 2 & 2 & 4 & -2 \\ 0 & 2 & 2 & 8 \\ 6 & 8 & 14 & 2 \end{bmatrix}$$

$$R_3 - 3R_1 \rightarrow R_3 \quad \begin{bmatrix} 2 & 2 & 4 & -2 \\ 0 & 2 & 2 & 8 \\ 0 & 2 & 2 & 8 \end{bmatrix} \quad R_3 - R_2 \rightarrow R_3 \quad \begin{bmatrix} 2 & 2 & 4 & -2 \\ 0 & 2 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$R_1 - R_2 \rightarrow R_1 \quad \begin{bmatrix} 2 & 0 & 2 & -10 \\ 0 & 2 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} \tfrac{1}{2}R_1 \rightarrow R_1 \\ \tfrac{1}{2}R_2 \rightarrow R_2 \end{matrix} \quad \begin{bmatrix} 1 & 0 & 1 & -5 \\ 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Question 2. *(10 marks)*
Consider the system whose augmented matrix is given in row echelon form below, where a is a constant:

$$\left[\begin{array}{cccc} 1 & 0 & 3 & 5 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & a^2-4 & a+2 \end{array}\right]$$

State the conditions on a so that the corresponding linear system has:
(i) No solutions
(ii) Infinitely many solutions
(iii) A unique solution

(i) $a^2-4=0$ & $a+2 \neq 0$
$a^2=4$
$a=\pm$ $\qquad$ $a \neq -2$
so $\boxed{a=2}$

(ii) $a^2-4=0$ & $a+2=0$
$a^2=4$
$a=\pm 2$ $\qquad$ $a=-2$
$\boxed{a=-2}$

(iii) $a^2-4 \neq 0$
$\boxed{a \neq \pm 2}$

Question 3. *(10 marks)*
Solve the following system of equations (note that there are 4 variables).
$x_1 - x_2 + 2x_3 = 1$
$2x_1 + 4x_3 - 2x_4 = 6$
$-2x_1 + 2x_2 - 4x_3 = -2$

$$\left[\begin{array}{cccc|c} 1 & -1 & 2 & 0 & 1 \\ 2 & 0 & 4 & -2 & 6 \\ -2 & 2 & -4 & 0 & -2 \end{array}\right] \begin{array}{l} -2R_1 + R_2 \to R_2 \\ R_3 + 2R_1 \to R_3 \end{array} \left[\begin{array}{ccccc} 1 & -1 & 2 & 0 & 1 \\ 0 & 2 & 0 & -2 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$\tfrac{1}{2} R_2 \to R_2 \quad \left[\begin{array}{ccccc} 1 & -1 & 2 & 0 & 1 \\ 0 & 1 & 0 & -1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

X_3 & X_4 are free variables

Let $X_3 = s$
$X_4 = t$

$x_2 - X_4 = 2$
$X_2 = 2 + X_4 = 2 + t$

$x_1 - X_2 + 2X_3 = 1$
$X_1 = 1 + X_2 - 2X_3$
$= 1 + 2 + t - 2s$
$= 3 - 2s + t$

$(x_1, x_2, x_3, x_4) = (3 - 2s + t, 2 + t, s, t)$

s, t in $\mathbb{R}$

Question 4. *(10 marks)*

$$A = \begin{bmatrix} 2 & -4 \\ 3 & 1 \\ -1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}$$

Compute the following (where possible).

(a) $3A + B^T$

(b) $CB - A^T$

(c) CA

(a) $3A + B^T$

$$= 3\begin{bmatrix} 2 & -4 \\ 3 & 1 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ -3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -12 \\ 9 & 3 \\ -3 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 6 & -11 \\ 10 & 3 \\ -6 & -2 \end{bmatrix}$$

(b) $CB - A^T$

$$= \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & -2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & -1 \\ -4 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 & 1 \\ 2 & 3 & -13 \end{bmatrix} - \begin{bmatrix} 2 & 3 & -1 \\ -4 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -2 & 2 \\ 6 & 2 & 3 \end{bmatrix}$$

(c) CA Not defined

$(2\times 2)(3\times 2)$

Question 5. *(10marks)*
Find A given the information below.

$$(A-2I)^{-1} = \begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}$$

$$((A-2I)^{-1})^{-1} = \begin{bmatrix} 2 & -3 \\ -4 & 2 \end{bmatrix}^{-1}$$

$$A - 2I = -\frac{1}{8}\begin{bmatrix} 2 & 3 \\ 4 & 2 \end{bmatrix}$$

$$A = \begin{bmatrix} -2/8 & -3/8 \\ -4/8 & -2/8 \end{bmatrix} + 2I$$

$$A = \begin{bmatrix} -1/4 & -3/8 \\ -1/2 & -1/4 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 7/4 & -3/8 \\ -1/2 & 7/4 \end{bmatrix}$$

Question 6. *(10 marks)*
Given n x n invertible matrices A and B. State whether or not the following statements are TRUE or FALSE. If the statement is false give an example otherwise simply state that the statement is true.
(a) $AB = BA$

FALSE $A = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$

$$AB = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 0 & 4 \end{bmatrix}$$

$$AB \neq BA$$

(b) $(A+B)^{-1} = A^{-1} + B^{-1}$

FALSE $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$(A+B)^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}^{-1} = \frac{1}{4}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

$$A^{-1} + B^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$(A+B)^{-1} \neq A^{-1} + B^{-1}$$

(c) $(AB)^{-1} = B^{-1}A^{-1}$

TRUE

Question 7. *(10 marks)*
(a) Given a square matrix A, state the two equations that a matrix C must satisfy in order for $C = A^{-1}$.
(b) Show that if a square matrix satifies $A^2 + 5A - I = 0$, then $A^{-1} = A + 5I$

(a) $CA = I$
$AC = I$

(b) Given $A^2+5A-I=0$
$I = A^2+5A$

MUST SHOW $A+5I$ is the inverse of A, so it must satisfy the two conditions stated in (a)

(1) $(A+5I)A = I$
(2) & $A(A+5I) = I$

(1) $(A+5I)A = A^2 + 5IA = A^2+5A = I$
FROM given
(2) $A(A+5I) = A^2 + A5I = A^2+5A = I$

The conditions ARE satisfied
so $A^{-1} = A+5I$ ■