LAST NAME: SOLUTIONS

FIRST NAME: ______________

STUDENT NUMBER: ______________

TEST 2 (B)

DAWSON COLLEGE

201-NYC-05 - Linear Algebra

Instructor: E. Richer

Date: July 3rd 2008

Question 1. *(10 marks)*

Solve the matrix equation. (Note: X is a matrix, not a scalar)

$$\begin{bmatrix} -1 & 3 \\ -1 & 0 \\ 2 & 4 \end{bmatrix} X = \begin{bmatrix} 8 & 4 \\ -1 & -2 \\ 14 & 12 \end{bmatrix}$$

X is A 2x2 MATRIX

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\begin{bmatrix} -1 & 3 \\ -1 & 0 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -a+3c & -b+3d \\ -a & -b \\ 2a+4c & 2b+4d \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ -1 & -2 \\ 14 & 12 \end{bmatrix}$$

By observation we get

$a=1$
$b=2$

$-a+3c = 8$
$-1+3c = 8$
$3c = 9$
$c = 3$

$-b+3d = 4$
$-2+3d = 4$
$3d = 6$
$d = 2$

CHECK

$$\begin{bmatrix} -1 & 3 \\ -1 & 0 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ -1 & -2 \\ 14 & 12 \end{bmatrix}$$

2B1

Question 2. *(10 marks)*
The following system of linear equations has a unique solution. Find the solution using the inverse of the coefficient matrix. (Note: No other method will earn you any points)

$x+y+z=4$
$x+y-3z=8$
$-3x+y+z=-4$

$$Ax = b$$

where $A=\begin{bmatrix}1&1&1\\1&1&-3\\-3&1&1\end{bmatrix}$ $x=\begin{bmatrix}x\\y\\z\end{bmatrix}$ $b=\begin{bmatrix}4\\8\\-4\end{bmatrix}$

$$A^{-1}: \left[\begin{array}{ccc|ccc}1&1&1&1&0&0\\1&1&-3&0&1&0\\-3&1&1&0&0&1\end{array}\right]$$

$R_2 - R_1 \rightarrow R_2$
$R_3 + 3R_1 \rightarrow R_3$
$$\left[\begin{array}{ccc|ccc}1&1&1&1&0&0\\0&0&-4&-1&1&0\\0&4&4&3&0&1\end{array}\right]$$
$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|ccc}1&1&1&1&0&0\\0&4&4&3&0&1\\0&0&-4&-1&1&0\end{array}\right]$$
$4R_1 \rightarrow R_1$
$R_2 + R_3 \rightarrow R_2$
$$\left[\begin{array}{ccc|ccc}4&4&4&4&0&0\\0&4&0&2&1&1\\0&0&-4&-1&1&0\end{array}\right]$$

$R_1 - R_2 \rightarrow R_1$
$$\left[\begin{array}{ccc|ccc}4&0&4&2&-1&-1\\0&4&0&2&1&1\\0&0&-4&-1&1&0\end{array}\right]$$
$R_1 + R_3 \rightarrow R_1$

$$\left[\begin{array}{ccc|ccc}4&0&0&1&0&-1\\0&4&0&2&1&1\\0&0&-4&-1&1&0\end{array}\right] \quad \left[\begin{array}{ccc|ccc}1&0&0&1/4&0&-1/4\\0&1&0&1/2&1/4&1/4\\0&0&1&1/4&-1/4&0\end{array}\right]$$

$$A^{-1}=\begin{bmatrix}1/4&0&-1/4\\1/2&1/4&1/4\\1/4&-1/4&0\end{bmatrix} \quad (x,y,z) = A^{-1}b = \begin{bmatrix}1/4&0&-1/4\\1/2&1/4&1/4\\1/4&-1/4&0\end{bmatrix}\begin{bmatrix}4\\8\\-4\end{bmatrix}$$

$\boxed{(x,y,z)=(2,3,-1)}$ $= \begin{bmatrix}2\\3\\\end{bmatrix}$

Question 3. *(10 marks)*

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

Find elementary matrices E_1, E_2 and E_3 that satisfy the following.

(i) $E_1 A = \begin{bmatrix} 2 & 2 & 2 \\ 1 & 1 & 1 \\ 3 & 3 & 3 \end{bmatrix}$

(ii) $E_2 A = \begin{bmatrix} -3 & -3 & -3 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

(ii) $E_3 A = \begin{bmatrix} 1 & 1 & 1 \\ -6 & -6 & -6 \\ 3 & 3 & 3 \end{bmatrix}$

(i) $R_1 \leftrightarrow R_2$ $\quad E_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(ii) $R_1 - 2R_2 \rightarrow R_1$ $\quad E_2 = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(iii) $-3R_2 \rightarrow R_2$ $\quad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Question 4. *(10 marks)*

Let $A = \begin{bmatrix} 1 & 1 & -2 & 1 \\ 2 & 4 & 0 & 1 \\ -2 & 4 & 2 & 2 \\ -3 & 0 & -4 & -1 \end{bmatrix}$

Evaluate the determinant of A by **reducing the matrix to row echelon form.**

$R_2 - 2R_1 \to R_2$
$R_3 + 2R_1 \to R_3$
$R_4 + 3R_1 \to R_4$

$\begin{bmatrix} 1 & 1 & -2 & 1 \\ 0 & 2 & 4 & -1 \\ 0 & 6 & -2 & 4 \\ 0 & 3 & -10 & 2 \end{bmatrix}$

$R_3 - 3R_2 \to R_3$ $\begin{bmatrix} 1 & 1 & -2 & 1 \\ 0 & 2 & 4 & -1 \\ 0 & 0 & -14 & 7 \\ 0 & 3 & -10 & 2 \end{bmatrix}$

$\frac{1}{2}R_2 \to R_2$
$-\frac{1}{14}R_3 \to R_3$

$\begin{bmatrix} 1 & 1 & -2 & 1 \\ 0 & 1 & 2 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 3 & -10 & 2 \end{bmatrix}$

$R_4 - 3R_2 \to R_4$ $\begin{bmatrix} 1 & 1 & -2 & 1 \\ 0 & 1 & 2 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & -16 & \frac{7}{2} \end{bmatrix}$

$R_4 + 16R_3 \to R_4$ $\begin{bmatrix} 1 & 1 & -2 & 1 \\ 0 & 1 & 2 & -\frac{1}{2} \\ 0 & 0 & 1 & -\frac{1}{2} \\ 0 & 0 & 0 & -\frac{9}{2} \end{bmatrix} = B$

$\det B = -\frac{9}{2}$

$\det B = \left(\frac{1}{2}\right)\left(-\frac{1}{14}\right) \det A$

$\det A = -\frac{9}{2}(2)(-14)$

$\boxed{= 126}$

Question 5. *(10 marks)*
Let A and B be 3x3 matrices with $\det A = 2$ and $\det B = -2$.
Compute the following.
(i) $\det(-2AB^T)$
(ii) $\det((3B)^{-1}(2A)^T)$

(i) $\det(-2AB^T)$

$= (-2)^3 \det A \det B^T$

$= (-2)^3 \det A \det B$

$= -8(2)(-2)$

$= \boxed{32}$

(ii) $\det\left((3B)^{-1}(2A)^T\right)$

$= \det\left(\frac{1}{3}B^{-1}2A^T\right)$

$= \det\left(\frac{2}{3}B^{-1}A^T\right)$

$= \left(\frac{2}{3}\right)^3 \det B^{-1} \det A^T$

$= \left(\frac{2}{3}\right)^3 \frac{1}{\det B} \det A$

$= \left(\frac{2}{3}\right)^3 \left(\frac{1}{-2}\right)(2)$

$= \boxed{-\left(\frac{2}{3}\right)^3}$

Question 6. *(10 marks)*
Given n x n matrices A and B. State whether or not the following statements are TRUE or FALSE. If the statement is false give an example otherwise simply state that the statement is true.
(a) $\det(AB) = \det(BA)$

TRUE

(b) $\det(A+B) = \det A + \det B$

FALSE

$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $A+B = \begin{bmatrix} 1 & 2 \\ 3 & -1 \end{bmatrix}$

$\det A = -3$ $\det B = -1$ $\det(A+B) = -7$

$\det A + \det B = -4$
$\det(A+B) = -7$ $\neq$

(c) $\det(2A) = 2\det A$

FALSE

$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ $\det A = -1$

$2A = \begin{bmatrix} 2 & 2 \\ 2 & 0 \end{bmatrix}$ $\det 2A = -4$

Question 8. *(10 marks)*

Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ where $\det A = 5$. Find $\det B$ where $B = \begin{bmatrix} a-g & b-h & c-i \\ 2d+2a & 2e+2b & 2f+2c \\ -4g & -4h & -4i \end{bmatrix}$.

$$B = \begin{bmatrix} a-g & b-h & c-i \\ 2d+2a & 2e+2b & 2f+2c \\ -4g & -4h & -4i \end{bmatrix} \xrightarrow{-\frac{1}{4}R_3 \to R_3} \begin{bmatrix} a-g & b-h & c-i \\ 2d+2a & 2e+2b & 2f+2c \\ g & h & i \end{bmatrix}$$

$$\xrightarrow{R_1+R_3 \to R_1} \begin{bmatrix} a & b & c \\ 2d+2a & 2e+2b & 2f+2c \\ g & h & i \end{bmatrix} \xrightarrow{R_2-2R_1 \to R_2} \begin{bmatrix} a & b & c \\ 2d & 2e & 2f \\ g & h & i \end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}R_2 \to R_2} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = A$$

$$\det A = -\frac{1}{4}\left(\frac{1}{2}\right)\det B$$

$$\det B = -4(2)\det A$$
$$= -8(5) = -40$$

$$\boxed{\det B = -40}$$

Question 7. *(10 marks)*
Prove that if A is an nxn matrix, then $\det(adjA) = (\det A)^{n-1}$

See version A SOLUTIONS
& CLASS NOTES