1A

LAST NAME: SOLUTIONS

FIRST NAME: ______________

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TEST 3 (A)

DAWSON COLLEGE

201-NYC-05 Linear Algebra

Instructor: E. Richer

Date: July 17th 2008

Question 1. *(10 marks)*

(a) Find a vector that is orthogonal to both $\vec{u} = (3,2,-1)$ and $\vec{v} = (1,-1,-1)$.

(b) Find the area of the parallelogram defined by $\vec{u}$ and $\vec{v}$.

(a) $\vec{u} \times \vec{v} = \boxed{(-3, 2, -5)}$

$\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$

(b) Area $= \|\vec{u} \times \vec{v}\|$

$= \sqrt{(-3)^2 + 2^2 + (-5)^2}$

$= \sqrt{9 + 4 + 25}$

$\boxed{= \sqrt{38}}$

Question 2. *(10 marks)*
(a) Explain briefly how the placement of the brackets in the scalar triple product $\vec{u}\cdot(\vec{v}\times\vec{w})$ matters.
(b) Find the volume of the parallelepiped defined by the vectors $\vec{u}=(2,-1,3)$, $\vec{v}=(1,-2,-1)$ and $\vec{w}=(4,-2,1)$

(a) $\vec{U}\cdot(\vec{V}\times\vec{W})$ is possible to compute
it is the dot product of TWO VECTORS

$(\vec{U}\cdot\vec{V})\times\vec{W}$ is impossible to compute it is cross product OF scalar & VECTOR

(b) $\left|\vec{U}\cdot(\vec{V}\times\vec{W})\right| = \text{volume}$

$$\underset{\begin{pmatrix}1\\-2\\-1\end{pmatrix}}{\vec{V}}\times\underset{\begin{pmatrix}4\\-2\\1\end{pmatrix}}{\vec{W}} = (-4,-5,6)$$

$$\left|\vec{U}\cdot(\vec{V}\times\vec{W})\right| = \left|(2,-1,3)\cdot(-4,-5,6)\right|$$
$$= \left|-8+5+18\right|$$
$$= |15| = \boxed{15}$$

Question 3. *(10 marks)*
Find an equation for the plane passing through the points $A(1,2,0)$, $B(-1,3,2)$ and $C(0,2,1)$.

$$\vec{AB} = (-2,1,2)$$
$$\vec{AC} = (-1,0,1)$$

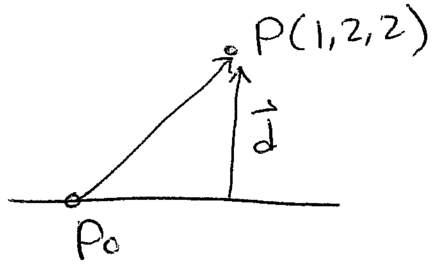
$$\vec{n} = \vec{AB} \times \vec{AC} = (1,0,1)$$
$$\begin{pmatrix}-2\\1\\2\end{pmatrix} \begin{pmatrix}-1\\0\\1\end{pmatrix}$$

$$x + z + d = 0$$

$$A(1,2,0) \rightarrow \quad 1+0+d=0$$
$$d=-1$$

$$\boxed{x+z-1=0}$$

Question 4. *(10 marks)*
Find the distance between the line $(x,y,z) = (1+t, -2-3t, 2t)$ and the point $P(1,2,2)$.



$P_0(1,-2,0)$ is on the line

$$\overrightarrow{P_0P} = (0,4,2)$$

$\vec{v}$ is the direction vector OF THE Line

$$\vec{v} = (1,-3,2)$$

$$\text{proj}_{\vec{v}} \overrightarrow{P_0P} = \left(\frac{\overrightarrow{P_0P}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v}$$

$$= \frac{-8}{14}(1,-3,2) = \left(\frac{-4}{7}, \frac{12}{7}, \frac{-8}{7}\right)$$

$$\vec{d} = \overrightarrow{P_0P} - \text{proj}_{\vec{v}} \overrightarrow{P_0P}$$
$$= (0,4,2) - (-4/7, 12/7, -8/7)$$
$$= (4/7, 16/7, 22/7) = 2/7(2, 8, 11)$$

$$\|\vec{d}\| = \frac{2}{7}\left(\sqrt{2^2+8^2+11^2}\right) = \boxed{\frac{2}{7}\sqrt{189}}$$

Question 5. *(10 marks)*
Find parametric equations for the line that is perpendicular to the plane $x-y+2z+2=0$ and passes through the point $P(1,2,3)$

$\vec{n} = (1,-1,2)$ this is the direction VECTOR FOR THE line

$(x,y,z) = (1,2,3) + (1,-1,2)t$

$\boxed{= (1+t,\ 2-t),3+2t)}$

Question 6. *(10 marks)*
Find an equation for the plane that is perpendicular to the plane $4x - y - z + 2 = 0$ and contains the line $(x, y, z) = (2 - 2t, 3t, 1 - t)$.

$\vec{n}_1 = (4,-1,-1)$ NORMAL, PARALLEL to PLANE we Are looking For

$\vec{d} = (-2,3,-1)$ diRECTION VECTOR OF Line

$\vec{n} = \vec{n}_1 \times \vec{d} = (4, 6, 10)$

$\begin{pmatrix} 4 \\ -1 \\ -1 \end{pmatrix} \begin{pmatrix} -2 \\ 3 \\ -1 \end{pmatrix}$

point on the plane (FROM the Line)
$P(2,0,1)$

$4x + 6y + 10z + d = 0$
$4(2) + 6(0) + 10(1) + d = 0$
$18 + d = 0 \quad d = -18$

$$\boxed{4x + 6y + 10z - 18 = 0}$$

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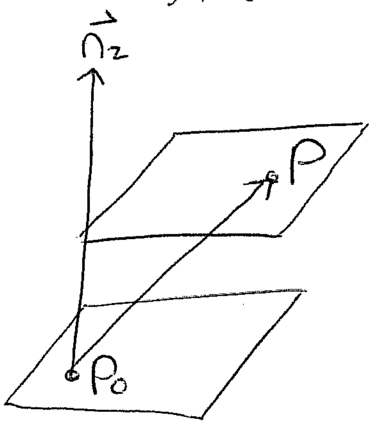
Question 7. *(10 marks)*
Find the distance between the two parallel planes $2x - y + z + 3 = 0$ and $4x - 2y + 2z - 3 = 0$.



P is on plane 1 $\quad P(0,3,0)$

P_0 is on plane 2 $\quad P_0(3/4, 0, 0)$

$$\vec{P_0P} = (-3/4, 3, 0)$$
$$\vec{n_2} = (4, -2, 2)$$

$$\text{proj}_{\vec{n_2}} \vec{P_0P} = \left(\frac{\vec{P_0P} \cdot \vec{n_2}}{\vec{n_2} \cdot \vec{n_2}}\right)\vec{n_2}$$
$$= \frac{-3-6}{24}(4,-2,2)$$
$$= -9/24\,(4,-2,2) = -9/12\,(2,-1,1)$$
$$= -3/4\,(2,-1,1)$$

$$\text{distance} = \|-3/4\,(2,-1,1)\|$$
$$= 3/4\sqrt{2^2+(-1)^2+1^2}$$
$$= 3/4\sqrt{6}$$

Question 8.

(a)*(5 marks)*

Show that the set of all 2x2 matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ with the standard matrix addition and scalar multiplication is **NOT** a vector space.

(b) *(10 marks)*

Let V be the set of all positive real numbers with operations

$x+y=xy$

$kx=x^k$

Prove that V satisfies axioms 4 and 5 for vector spaces (see back page for vector space axioms).

(a) AXIOM 1 FAILS

if $u=\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ & $v=\begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix}$ are in V

then $u+v=\begin{bmatrix} a+c & 2 \\ 2 & b+d \end{bmatrix}$ is Not in V

(b) AXIOM 4

THE $0_V = 1$

then $x+0_V = x+1 = x \cdot 1 = x$

AXIOM 5

$-x = \frac{1}{x}$

then $x+(-x) = x\left(\frac{1}{x}\right) = 1 = 0_V$

BONUS *(5 marks)*
Let $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$. Prove that $\vec{u}$ is orthogonal to $\vec{u} \times \vec{v}$.

$$\vec{U} \times \vec{V} = (U_2V_3 - V_2U_3, -(U_1V_3 - V_1U_3), U_1V_2 - V_1U_2)$$

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix}$$

$$\vec{U} \cdot (\vec{U} \times \vec{V}) = U_1U_2V_3 - U_1V_2U_3 - U_2U_1V_3 + U_2V_1U_3 + U_3U_1V_2 - U_3V_1U_2$$

$$= 0$$

So $\vec{U}$ & $\vec{U} \times \vec{V}$ ARe orthogonAl.

Vector Space Axioms

1- If u and v are objects in V, then $u+v$ is in V.

2- $u+v=v+u$

3- $u+(v+w)=(u+v)+w$

4- There is an object 0_v called a zero object for V such that $0_v+u=u$ for all u in V.

5- For each u in V, there is an object $-u$ in V called a negative of u such that $u+(-u)=0_v$

6- If k is any scalar and u is any object in V, then ku is in V

7- k(u+v) = ku + kv

8- (k+m)u = ku + mu

9- k(mu) = (km)u

10- 1u = u