LAST NAME: SOLUTIONS

FIRST NAME: ______________

STUDENT NUMBER: ______________

TEST 3 (B)

DAWSON COLLEGE

201-NYC-05 Linear Algebra

Instructor: E. Richer

Date: July 17th 2008

Question 1. *(10 marks)*
(a) Find a vector that is orthogonal to both $\vec{u} = (1,2,-4)$ and $\vec{v} = (2,-1,-1)$.
(b) Find the area of the parallelogram defined by $\vec{u}$ and $\vec{v}$.

(a) $\vec{u} \times \vec{v} = \boxed{(-6,-7,-5)}$

$\begin{pmatrix} 1 \\ 2 \\ -4 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$

(b) Area $= \|\vec{u} \times \vec{v}\| = \sqrt{(-6)^2+(-7)^2+(-5)^2}$

$= \boxed{\sqrt{110}}$

Question 2. *(10 marks)*
(a) Explain briefly how the placement of the brackets in the scalar triple product $\vec{u}\cdot(\vec{v}\times\vec{w})$ matters.
(b) Find the volume of the parallelepiped defined by the vectors $\vec{u} = (1,-1,3)$, $\vec{v} = (2,-2,-1)$ and $\vec{w} = (4,-2,1)$

(a) $\vec{U}\cdot(\vec{V}\times\vec{W})$

is the dot product of 2 vectors, permissible operation

$(\vec{U}\cdot\vec{V})\times\vec{W}$

is a scalar crossed with a vector, impossible

(b) $|\vec{U}\cdot\vec{V}\times\vec{W}|$

$= \left|\vec{U}\cdot\begin{pmatrix}2\\-2\\-1\end{pmatrix}\times\begin{pmatrix}4\\-2\\1\end{pmatrix}\right|$

$= |\vec{U}\cdot(-4,-6,4)|$

$= |(1,-1,3)\cdot(-4,-6,4)|$

$= |-4+6+12| = \boxed{14}$

Question 3. *(10 marks)*
Find an equation for the plane passing through the points $A(0,2,2)$, $B(-1,-2,2)$ and $C(-1,3,1)$.

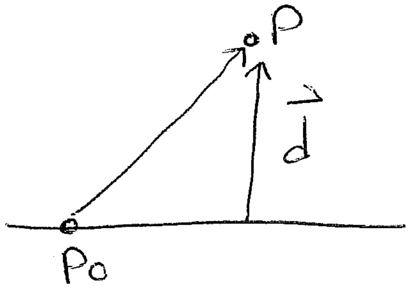
$$\vec{AB} = (-1,-4,0)$$
$$\vec{AC} = (-1,1,-1)$$

$$\vec{n} = \vec{AB} \times \vec{AC} = (4,-1,-5)$$
$$\begin{pmatrix}-1\\-4\\0\end{pmatrix} \begin{pmatrix}-1\\1\\-1\end{pmatrix}$$

$$4x - y - 5z + d = 0$$
$$4(0) - 2 - 5(2) + d = 0$$
$$-12 + d = 0$$
$$d = 12$$

$$\boxed{4x - y - 5z + 12 = 0}$$

Question 4. *(10 marks)*
Find the distance between the line $(x,y,z) = (2t, -1-3t, 1+t)$ and the point $P(1,1,2)$.



$P_0\ (0,-1,1)$ is on the line

$$\overrightarrow{P_0P} = (1,2,1)$$

$\vec{v} = (2,-3,1)$ direction vector of Line

$$\text{Proj}_{\vec{v}}\, \overrightarrow{P_0P} = \left(\frac{\overrightarrow{P_0P}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\right)\vec{v}$$

$$= \frac{(2-6+1)}{4+9+1}(2,-3,1)$$

$$= \frac{-3}{14}(2,-3,1)$$

$$\vec{d} = \overrightarrow{P_0P} - \text{proj}_{\vec{v}}\, \overrightarrow{P_0P}$$

$$= (1,2,1) + \frac{3}{14}(2,-3,1)$$

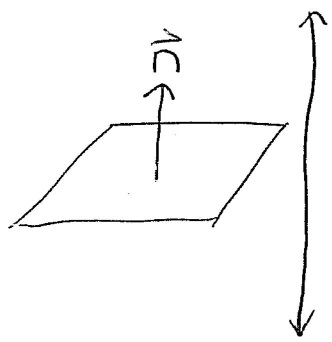
$$= \left(\frac{20}{14}, \frac{19}{14}, \frac{17}{14}\right)$$

$$\|\vec{d}\| = \frac{1}{14}\sqrt{20^2+19^2+17^2} = \boxed{\frac{1}{14}\sqrt{1050}}$$

1B

Question 5. *(10 marks)*
Find parametric equations for the line that is perpendicular to the plane $2x+y+2z+2=0$ and passes through the point $P(1,2,2)$

$\vec{n} = (2,1,2)$
is A direction
vector for the line



$(x,y,z) = (1,2,2) + (2,1,2)t$
$\boxed{= (1+2t,\ 2+t,\ 2+2t)}$

Question 6. *(10 marks)*
Find an equation for the plane that is perpendicular to the plane $2x - y - 4z + 2 = 0$ and contains the line $(x, y, z) = (2 - t, 2t, 1 - t)$.

$\vec{n}_1 = (2, -1, -4)$ Normal of plane that is perpendicular to the plane we are looking for

$\vec{d} = (-1, 2, -1)$ direction vector of line

$\vec{n} = \vec{n}_1 \times \vec{d} = (9, 6, 3)$

$\begin{pmatrix} 2 \\ -1 \\ -4 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$

$9x + 6y + 3z + d = 0$

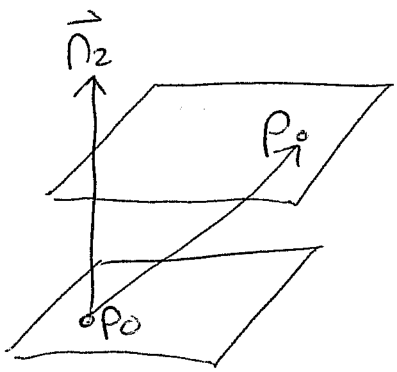
$P(2, 0, 1)$ is on the line, so on the plane

$9(2) + 6(0) + 3(1) + d = 0$

$18 + 3 + d = 0 \quad d = -21$

$$\boxed{9x + 6y + 3z - 21 = 0}$$

Question 7. *(10 marks)*
Find the distance between the two parallel planes $2x-y+z+3=0$ and $4x-2y+2z+7=0$



P is on plane 1
$P(0,0,-3)$

P_0 is on plane 2

$P_0(-7/4,0,0)$

$\vec{P_0P} = (7/4, 0, -3)$

$\vec{n_2} = (4,-2,2)$

$$\text{Proj}_{\vec{n_2}} \vec{P_0P} = \left(\frac{(7/4,0,-3)\cdot(4,-2,2)}{(4,-2,2)\cdot(4,-2,2)}\right)(4,-2,2)$$

$$= \frac{1}{24}(4,-2,2)$$

$$\text{distance} = \left\| \frac{1}{24}(4,-2,2) \right\|$$

$$\boxed{= \frac{1}{24}\sqrt{24}}$$

Question 8.

(a)*(5 marks)*

Show that the set of all 2x2 matrices of the form $\begin{bmatrix} a & b \\ 2 & 1 \end{bmatrix}$ with the standard matrix addition and scalar multiplication is **NOT** a vector space.

(b) *(10 marks)*

Let V be the set of all pairs of real numbers $(1,x)$ with operations defined as follows:

$(1,x)+(1,y) = (1,x+y)$

$k(1,x) = (1,kx)$

Prove that V satisfies axioms 4 and 5 for vector spaces (see back page for vector space axioms).

(a) AXIOM 1 FAILS

Let $U = \begin{bmatrix} a & b \\ 2 & 1 \end{bmatrix}$ $V = \begin{bmatrix} c & d \\ 2 & 1 \end{bmatrix}$ be in V

$U+V = \begin{bmatrix} a+c & b+d \\ 4 & 2 \end{bmatrix}$ Not in V

(b) AXIOM 4

THE $0_V = (1,0)$

then $U = (1,x)$

$U + 0_V = (1,x)+(1,0) = (1, x+0)$
$= (1,x) = U$

Axiom 5

$U = (1,x)$ $-U = (1,-x)$

then $U+(-U) = (1,x) + (1,-x)$
$= (1, x-x) = (1,0) = 0_V$

1B

BONUS *(5 marks)*
Let $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$. Prove that $\vec{u}$ is orthogonal to $\vec{u} \times \vec{v}$.

See Version a
or class Notes

Vector Space Axioms

1- If u and v are objects in V, then $u+v$ is in V.

2- $u+v=v+u$

3- $u+(v+w)=(u+v)+w$

4- There is an object 0_v called a zero object for V such that $0_v+u=u$ for all u in V.

5- For each u in V, there is an object $-u$ in V called a negative of u such that $u+(-u)=0_v$

6- If k is any scalar and u is any object in V, then ku is in V

7- k(u+v) = ku + kv

8- (k+m)u = ku + mu

9- k(mu) = (km)u

10- 1u = u