Simple Interest

October 26, 2008

Simple Interest:

- *Interest* is a fee charged for the borrowing of capital (a rent on money). *Simple interest* is governed by the following quation.

$$\text{Interest} = \text{Principal} \times \text{Rate} \times \text{Time}$$
$$I = Prt$$

where I is the amount of interest accumulated after t time (*interest period*, the time unit is measured in years) at the rate r (percentage per year) for the P principal sum of money.

- Simple interest is used in short-term loans, savings bonds and purchase on credit.

Simple Interest: Example

- Compute the interest on the amount of $2 847.88 at a rate of 7.6% over 122 days.

$$
\begin{aligned}
I &= Prt \\
&= 2847.88(0.076)\left(\frac{122}{365}\right) \\
&= \$72.34
\end{aligned}
$$

- Compute the interest for the amount of $5 321.23 at a rate of 12.5% over two months.

$$
\begin{aligned}
I &= Prt \\
&= 5\,321.23(0.125)\left(\frac{2}{12}\right) \\
&= \$110.86
\end{aligned}
$$

Simple Interest: Finding P, r, or t

- If the interest I is given, and two other variables are known one can isolate the unknown variable. Using values of the variables one can determine the unknown value of the variable.

$$I = Prt$$

Isolating the principal:

$$P = \frac{I}{rt}$$

Isolating the rate:

$$r = \frac{I}{Pt}$$

Isolating the time:

$$t = \frac{I}{Pr}$$

Simple Interest: Finding P, r, or t: Examples

1. What rate of interest will earn $43.23 if the principal of $2 040.00 is invested for 129 days?

$$I = Prt$$
$$r = \frac{I}{Pt}$$
$$r = \frac{43.23}{2040.00\left(\frac{129}{365}\right)}$$
$$r = 6\%$$

2. What time is required to obtain $19.34 if the principal of $1 746.33 is invested at 3.4%?

$$I = Prt$$
$$t = \frac{I}{Pr}$$
$$t = \frac{19.34}{1746.33(0.034)}$$
$$t = 0.3257 \text{ year}$$
$$t = 119 \text{ days}$$

Simple Interest: Finding P, r, or t: Examples

1. What principal amount needs to be invested to earn $431.23 if the principal is invested for 2 years at a rate of 3.75%?

$$
\begin{aligned}
I &= Prt \\
P &= \frac{I}{rt} \\
P &= \frac{431.23}{0.0375(2)} \\
P &= \$5\,749.73
\end{aligned}
$$

Simple Interest: Future Value(Maturity Value)

- *Future value* is the sum of the principal and the interest.

$$\text{Future Value} = \text{Principal} + \text{Interest}$$
$$S = P + I$$

- The above formula can be combined with the equation for simple interest to obtain:

$$S = P + I$$
$$S = P + Prt$$
$$S = P(1 + rt)$$

- **Example:** Emma invested \$10 462.21 for $5\frac{1}{2}$ years at $4\frac{1}{4}\%$. Determine the maturity value of the investment?

$$S = P(1 + rt)$$
$$= 10\,462.21(1 + 0.0425(5.5))$$
$$= \$12\,907.75$$

Simple Interest: Present Value

- *Present value* is the required principal needed to obtain a future value. The equation is obtained from the future value equation $S = P(1+rt)$ and isolating P:
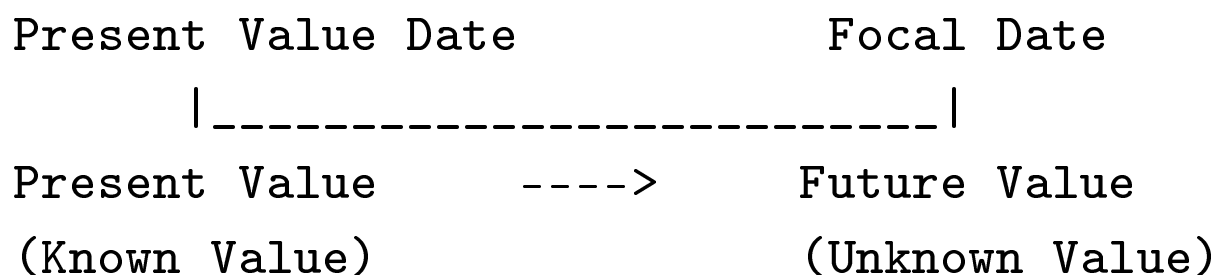
$$P = \frac{S}{1+rt}$$

- **Example:** What principal is required to have a future value of $\$10\,000$ in 5 years if the interest rate is 3.25%?

$$
\begin{aligned}
P &= \frac{S}{1+rt} \\
&= \frac{10000}{1+0.0325(5)} \\
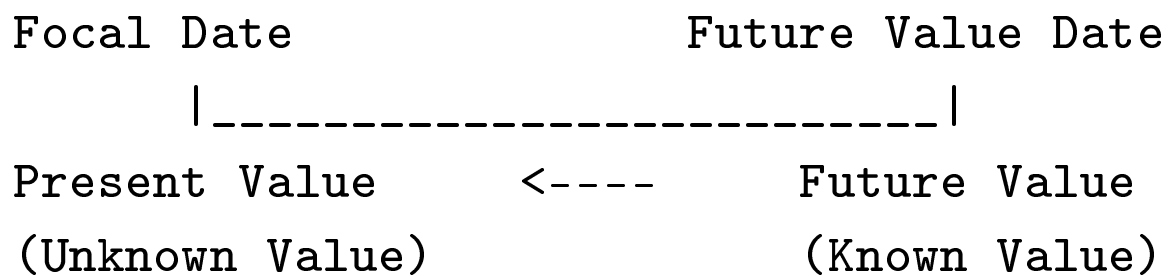&= \$8\,602.15
\end{aligned}
$$

Simple Interest: Equivalent Values

- Money subject to interest will grow over time. The value of money at a given time is called the *equivalent value*. A method to compare the value of money is to compare its value at a chosen date called the *focal date*.

- If the focal date is after the present value date we use the formula $S = P(1 + rt)$ to determine the equivalent value at a future time.

```
Present Value Date                Focal Date
      |_____|
Present Value       ---->       Future Value
(Known Value)                  (Unknown Value)
```

It follows that the future value will be greater than the known principal.

- If the focal date is before the future value date we use the formula $P = \frac{S}{1+rt}$ to determine the equivalent value at an earlier time.

```
Focal Date                      Future Value Date
       |_____|
Present Value        <----        Future Value
(Unknown Value)                   (Known Value)
```

It follows that the present value will be less than the future value.

Simple Interest: Equivalent Values: Example: Single Payment

- The Agrarian Bike Shop was owed a payment 9 months ago of $450.67 at 9.8% and is owed a payment of $1 200.00 in 5 months at 9.8%. Instead the Agrarian Bike Shop will be given a single equivalent payment today. What is the amount of the single payment?

```
9 months                    Today      5 months
    |_____|_____|
$450.67        ---->        | <---- $1 200.00
                    Focal Date
```
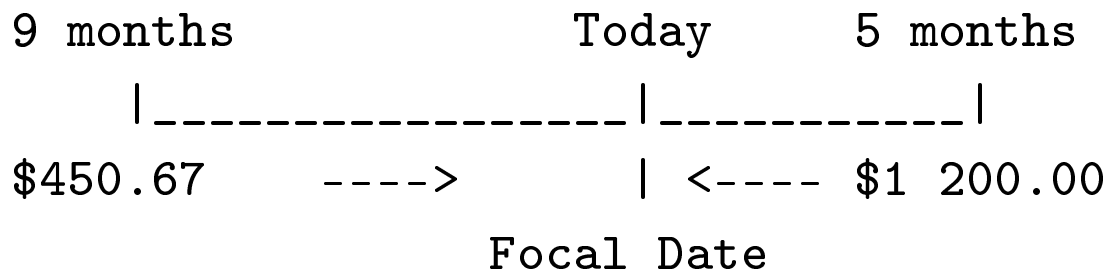
The equivalent value of $450.67, today is

$$
\begin{aligned}
S &= P(1+rt) \\
&= 450.67\left(1+0.098\left(\frac{9}{12}\right)\right) \\
&= \$483.79
\end{aligned}
$$

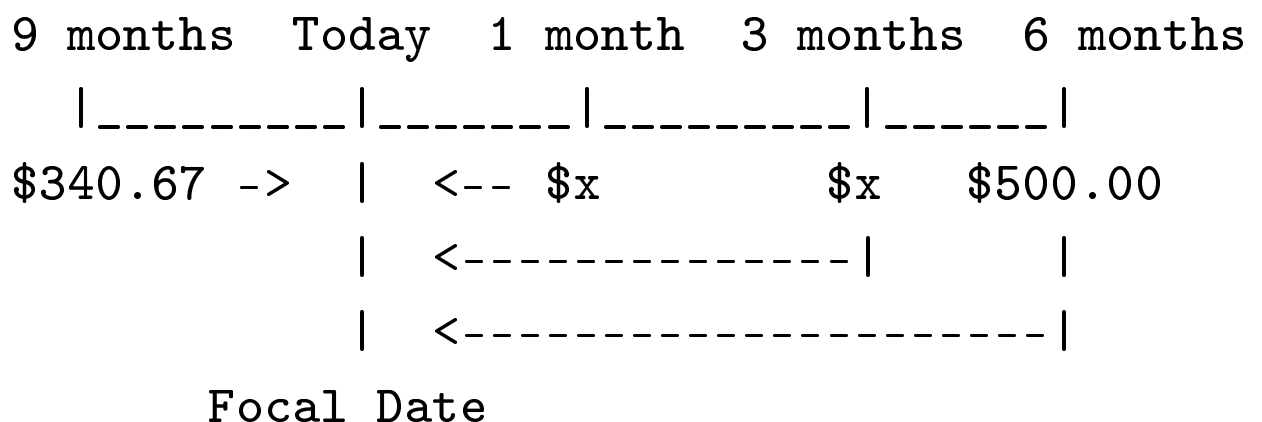The equivalent value of $1\ 200.00$, today is

$$P = \frac{S}{1+rt}$$

$$= \frac{1200}{1+0.098\left(\frac{5}{12}\right)}$$

$$= \$1\ 152.92$$

Therefore the amount of the single payment is $483.79 + 1\ 152.92 = \$1\ 636.71$

Simple Interest: Equivalent Values: Example: Equal Payments

- The Agrarian Bike Shop owed a payment 9 months ago of $340.67 at 12.5% and owes a payment of $500.00 in 6 months at 11%. Instead the Agrarian Bike Shop will repay the two debts in two equal sized payment. One payment in one month and a second payment in 3 months. Money is now worth 10.5%. What is the size of the equal payments?

```
9 months  Today  1 month  3 months  6 months
  |_____|_____|_____|_____|
$340.67 ->  |   <-- $x           $x    $500.00
            |   <--------------|        |
            |   <---------------------|
        Focal Date
```

where x is the size of the equal payments. The equiva-

lent value of $340.67 at 12%, today is

$$
\begin{aligned}
S & = P(1+rt) \\
& = 340.67\left(1+0.125\left(\frac{9}{12}\right)\right) \\
& = \$372.61
\end{aligned}
$$

The equivalent value of $500.00 at 11%, today is

$$
\begin{aligned}
P & = \frac{S}{1+rt} \\
& = \frac{500}{1+0.11\left(\frac{6}{12}\right)} \\
& = \$473.93
\end{aligned}
$$

The equivalent values of x in 1 month and 3 months at 10.5%, respectively, is

$$
\begin{aligned}
P & = \frac{S}{1+rt} \\
& = \frac{x}{1+0.105\left(\frac{1}{12}\right)} \\
& = 0.991325898x
\end{aligned}
$$

$$P = \frac{S}{1+rt}$$

$$= \frac{x}{1+0.105\left(\frac{3}{12}\right)}$$

$$= 0.974421437x$$

To solve for x we set the replacement payments on one side of the equation and the original payments on the other. We obtain

$$0.991325898x + 0.974421437x = 372.61 + 473.93$$
$$1.965747335x = 846.54$$
$$x = \$430.65$$

Therefore the size of the equal payments are $430.65.