

SOLUTIONS - Extra Related Rates problems

(Q1) Given $\frac{dv}{dt} = 1 \text{ cm}^3/\text{s}$

Looking for $\frac{dd}{dt}$ when $d = 10 \text{ cm}$
($d = \text{diameter of sphere}$)

$$V = \frac{4}{3} \pi r^3 \quad \text{but} \quad r = \frac{d}{2} \quad \left(\text{radius} = \frac{\text{diameter}}{2} \right)$$

so $V = \frac{4}{3} \pi \left(\frac{d}{2} \right)^3$

$$V = \frac{4}{24} \pi d^3$$

$$V = \frac{\pi}{6} d^3$$

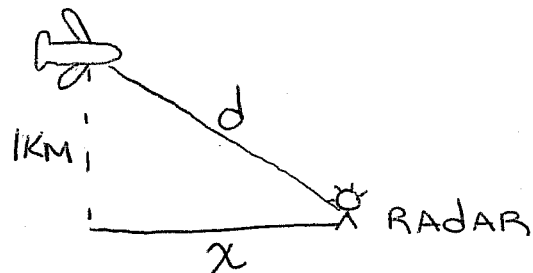
DIFFERENTIATE WITH RESPECT TO t)

$$\frac{dv}{dt} = \frac{\pi}{2} d^2 \frac{dd}{dt}$$

$$1 = \frac{\pi}{2} (10)^2 \frac{dd}{dt}$$

$$\boxed{\frac{dd}{dt} = 1/50 \pi \text{ cm/s}}$$

(Q2)



d = distance between radar station & plane

$$\frac{dx}{dt} = 500 \text{ km/h}$$

Looking for $\frac{dd}{dt}$ when $d = 2 \text{ km}$

Using Pythagorean Theorem

$$1^2 + x^2 = d^2$$

differentiate with respect to t

$$2x \frac{dx}{dt} = 2d \frac{dd}{dt}$$

Find x when $d = 2$

$$1^2 + x^2 = 2^2$$

$$x^2 = 3$$

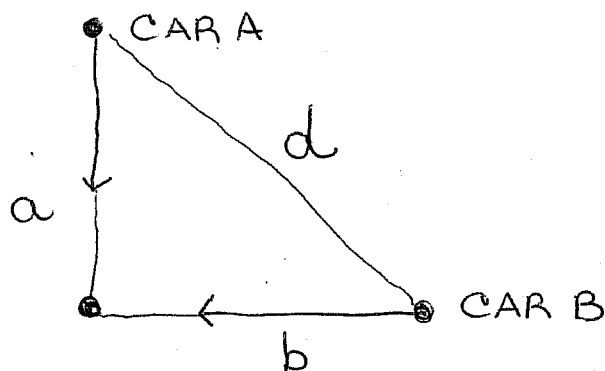
$$x = \sqrt{3}$$

$$2\sqrt{3}(500) = 2(2) \frac{dd}{dt}$$

$$\frac{dd}{dt} = \frac{1000\sqrt{3}}{4}$$

$$\boxed{\frac{dd}{dt} = 250\sqrt{3} \text{ km/h}}$$

(Q3)



CAR a: $\frac{da}{dt} = 60 \text{ km/h}$

$$\frac{db}{dt} = 25 \text{ km/h}$$

Looking for $\frac{dd}{dt}$ 2 HOURS LATER

2 HOURS LATER

CAR A HAS TRAVELED 120 km

CAR B HAS TRAVELED 50 km

So $a = 120$ & $b = 50$ & $a^2 + b^2 = d^2$
 $120^2 + 50^2 = d^2$
 $d = 130$

$$a^2 + b^2 = d^2$$

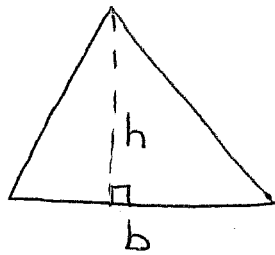
$$2a \frac{da}{dt} + 2b \frac{db}{dt} = 2d \frac{dd}{dt}$$

$$2(120)(60) + 2(50)25 = 2(130) \frac{dd}{dt}$$

$$\frac{14400 + 2500}{260} = \frac{dd}{dt}$$

$$\boxed{\frac{dd}{dt} = 65 \text{ km/h}}$$

(Q4)



$$\frac{dh}{dt} = 1 \text{ cm/min}$$

$$\frac{dA}{dt} = 2 \text{ cm}^2/\text{min}$$

Looking for $\frac{db}{dt}$ when $h=10$
& $A=100$

$$A = \frac{1}{2} b h$$

$$\frac{dA}{dt} = \frac{1}{2} \frac{db}{dt} h + \frac{1}{2} \frac{dh}{dt} b \quad (\text{PRODUCT RULE})$$

when $A=100$ & $h=10$

$$100 = \frac{1}{2} b (10)$$

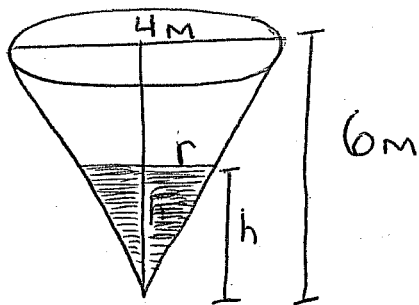
$$b = 20$$

$$2 = \frac{1}{2} \frac{db}{dt} (10) + \frac{1}{2} (1) (20)$$

$$-8 = 5 \frac{db}{dt}$$

$$\boxed{\frac{db}{dt} = -\frac{8}{5} \text{ cm/min}}$$

(Q5)



$$\frac{dh}{dt} = 0.2$$

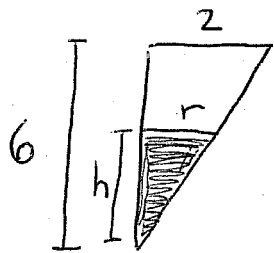
when
 $h = 2\text{m}$

Looking for dV/dt

V = volume of water in tank

$$V = \frac{1}{3} \pi r^2 h$$

Express r in terms of h



$$\frac{r}{h} = \frac{2}{6}$$

$$r = h/3$$

$$V = \frac{1}{3} \pi \left(\frac{h}{3}\right)^2 h$$

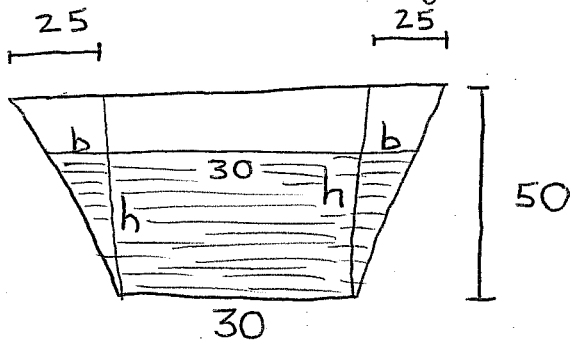
$$V = \frac{\pi}{27} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{9} h^2 \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{\pi}{9} (2)^2 (0.2)$$

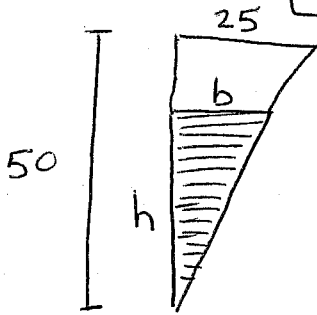
$$\boxed{\frac{dV}{dt} = \frac{0.8 \text{ m}^3}{9 \text{ min}}}$$

(Q6) Looking for $\frac{dh}{dt}$ when $h=30$



Given $\frac{dV}{dt} = 20$

$$V = (\text{AREA OF TRAPEZOID}) \times \text{length of trough}$$
$$= \left[30h + 2\left(\frac{1}{2}bh\right) \right] \times 1000$$



$$\frac{b}{h} = \frac{25}{50}$$

$$b = \frac{h}{2}$$

$$V = \left[30h + 2\left(\frac{1}{2}\left(\frac{h}{2}\right)h\right) \right] \times 1000$$

$$V = 30000h + 500h^2$$

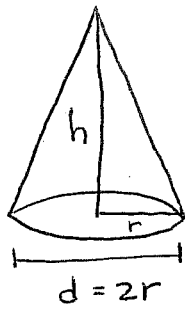
$$\frac{dV}{dt} = 30000\frac{dh}{dt} + 1000h\frac{dh}{dt}$$

$$20 = (30000 + 1000(30))\frac{dh}{dt}$$

$$20 = 60000\frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{1}{3000} \text{ cm/min}}$$

(Q7)



Looking for

$\frac{dh}{dt}$ when $h=10$

Given $\frac{dV}{dt} = 30$

& $d = h$

$2r = h$

$r = h/2$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{h}{2}\right)^2 h$$

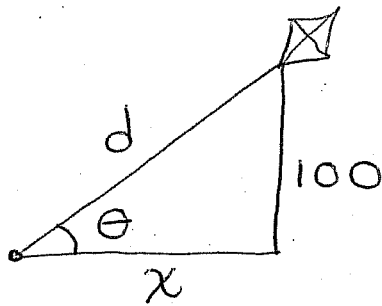
$$V = \frac{\pi}{12} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$$

$$30 = \frac{\pi}{4} (10)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{120}{100\pi} = \frac{6}{5\pi} \text{ ft/s}$$

(Q8)



Looking for $\frac{d\theta}{dt}$ when $d = 200$

when $d = 200$

$$x^2 + 100^2 = 200^2$$

$$x^2 = 200^2 - 100^2$$

$$x^2 = 40000 - 10000$$

$$x^2 = 30000$$

$$x = \sqrt{30000}$$
$$= 100\sqrt{3}$$

Given $\frac{dx}{dt} = 200$

$$\tan \theta = \frac{100}{x}$$

$$\sec^2 \theta \frac{d\theta}{dt} = 100 (-x^{-2}) \frac{dx}{dt}$$

when $d = 200$ & $x = 100\sqrt{3}$

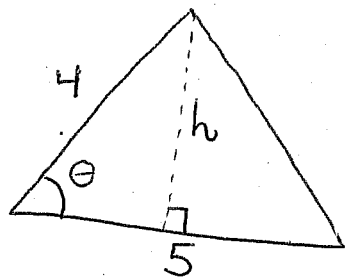
$$\cos \theta = \frac{100\sqrt{3}}{200} = \frac{\sqrt{3}}{2}$$

$$\sec \theta = \frac{2}{\sqrt{3}}$$

$$\left(\frac{2}{\sqrt{3}}\right)^2 \frac{d\theta}{dt} = \frac{-100}{(100\sqrt{3})^2} (8)$$

$$\boxed{\frac{d\theta}{dt} = \frac{-8}{300} \left(\frac{3}{4}\right) = \frac{-1}{50} \text{ ft/s}}$$

(Q9)



Looking for

$$\frac{dA}{dt} \text{ when } \theta = \pi/3$$

$$\text{Given } d\theta/dt = 0.06$$

$$A = \frac{bh}{2}$$

$$A = \frac{5h}{2}$$

We want to express AREA in terms of the variable θ

$$\sin \theta = \frac{h}{4}$$

$$h = 4 \sin \theta$$

$$A = \frac{5(4 \sin \theta)}{2}$$

$$A = 10 \sin \theta$$

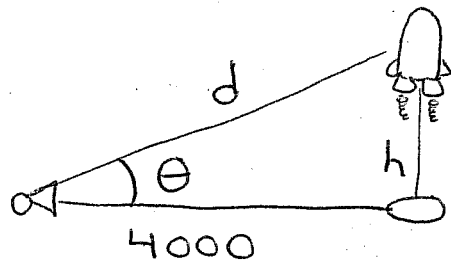
$$dA/dt = 10 \cos \theta d\theta/dt$$

$$dA/dt = 10 \cos(\pi/3) 0.06$$

$$dA/dt = 10 \left(\frac{1}{2}\right) 0.06$$

$$\boxed{dA/dt = 0.3 \text{ m}^2/\text{s}}$$

(Q10)



(a) Looking for dd/dt when $h=3000$

Given $dh/dt = 600$

$$(4000)^2 + h^2 = d^2$$

$$2h dh/dt = 2d dd/dt$$

When $h=3000$

$$(4000)^2 + (3000)^2 = d^2$$

$$d = 5000$$

$$2(3000)(600) = 2(5000) dd/dt$$

$$dd/dt = 360 \text{ ft/s}$$

(b) Looking for $d\theta/dt$ when $h=3000$

$$\tan \theta = \frac{h}{4000}$$

$$\sec^2 \theta d\theta/dt = \frac{1}{4000} dh/dt$$

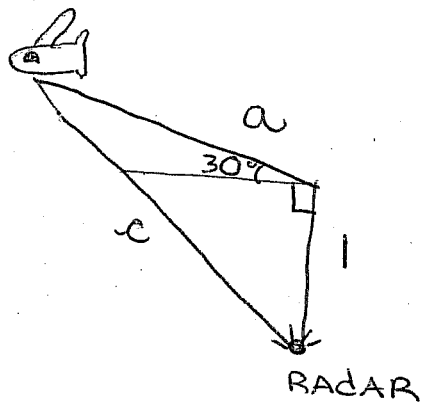
$$\cos \theta = \frac{4000}{5000} = 4/5$$

$$\sec \theta = 5/4$$

$$(5/4)^2 d\theta/dt = \frac{1}{4000} (600)$$

$$d\theta/dt = \frac{12}{125} \text{ rad/s}$$

(Q11)



Looking for
 $\frac{dc}{dt}$ 1 min later

AFTER 1 min

$$a = \frac{300}{60} \\ = 5 \text{ km}$$

$$\text{Given } \frac{da}{dt} = 300$$

Using the Law of Cosines
we can find c one minute later.

$$c^2 = a^2 + 1^2 - 2a \cos(120^\circ)$$

$$c^2 = (5)^2 + 1^2 - 2(5)(-\frac{1}{2})$$

$$c = \sqrt{31}$$

$$c^2 = a^2 + 1^2 - 2a \cos(120^\circ)$$

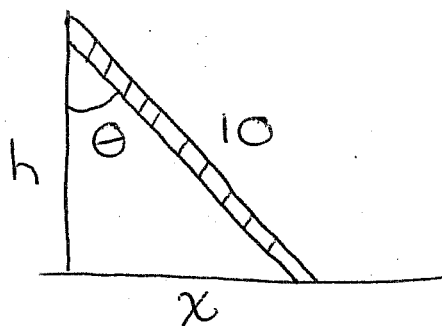
$$c^2 = a^2 + 1^2 + a$$

$$2c \frac{dc}{dt} = 2a \frac{da}{dt} + \frac{da}{dt}$$

$$2\sqrt{31} \frac{dc}{dt} = 2(5)(300) + 300$$

$$\frac{dc}{dt} = \frac{3300}{2\sqrt{31}} = \frac{1650}{\sqrt{31}} \text{ Km/hr}$$

(Q12)



Looking for $\frac{d\theta}{dt}$ when $\theta = \pi/4$

Given $\frac{dx}{dt} = 2$

$$\sin \theta = \frac{x}{10}$$

$$\cos \theta \frac{d\theta}{dt} = \frac{1}{10} \frac{dx}{dt}$$

$$\cos(\pi/4) \frac{d\theta}{dt} = \frac{1}{10} (2)$$

$$\frac{d\theta}{dt} = \frac{1}{5} \left(\frac{\sqrt{2}}{1} \right)$$

$$\boxed{\frac{d\theta}{dt} = \frac{\sqrt{2}}{5} \text{ rad/s}}$$