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Quiz #2 since the first course by Yann

This quiz is graded out of 10 marks. No books, graphing calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §8.8#20 *(10 marks)*
Determine whether the improper integral diverges or converges. Evaluate the integral if it converges.

$$\int_0^\infty xe^{-x/2}\,dx$$

$$= \lim_{b\to\infty} \int_0^b xe^{-x/2}\,dx$$ integrate by parts

$u = x \quad du = dx$

$v = -2e^{-x/2} \quad dv = e^{-x/2}dx$

$$= \lim_{b\to\infty} \left[uv\Big|_0^b - \int_0^b v\,du \right]$$

$$= \lim_{b\to\infty} \left[-2xe^{-x/2}\Big|_0^b + 2\int_0^b e^{-x/2}dx \right]$$

$$= \lim_{b\to\infty} \left[-2be^{-b/2} + 2(0)e^0 \; -4e^{-x/2}\Big|_0^b \right]$$

$$= \lim_{b\to\infty} \left[-2be^{-b/2} - 4e^{-b/2} + 4e^0 \right]$$

$$= \lim_{b\to\infty} \left[\frac{-2b}{e^{b/2}} - \frac{4}{e^{b/2}} + 4 \right]$$ ($e^0 = 0$)

$$= 4 + \lim_{b\to\infty} \frac{-2b}{e^{b/2}}$$ the indeterminate form is $\frac{-\infty}{\infty}$ ∴ we use L'hopital's rule

$$= 4 + \lim_{b\to\infty} \frac{\frac{d}{db}[-2b]}{\frac{d}{db}[e^{b/2}]}$$

$$= 4 + \lim_{b\to\infty} \frac{-2}{\frac{e^{b/2}}{2}} \qquad = 4 + \lim_{b\to\infty} \frac{-4}{e^{b/2}} \quad (e^0 = 0) \qquad = 4$$