

Quiz #3 since the first course by Yann

This quiz is graded out of 10 marks. No books, graphing calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §9.1#65 (7 marks) Determine whether the sequence diverges or converges. Find the limit if it converges.

$$a_n = \left(1 + \frac{k}{n}\right)^n$$

notice that $\lim_{n \rightarrow \infty}$ has indeterminate for 1^∞

Suppose $y = \lim_{n \rightarrow \infty} \left(1 + \frac{k}{n}\right)^n$

$$\ln y = \ln \left(\lim_{n \rightarrow \infty} \left(1 + \frac{k}{n}\right)^n \right)$$

$$\ln y = \lim_{n \rightarrow \infty} \ln \left(1 + \frac{k}{n}\right)^n$$

$$\ln y = \lim_{n \rightarrow \infty} n \ln \left(1 + \frac{k}{n}\right) \quad \text{has IF } \infty \cdot 0$$

$$\ln y = \lim_{n \rightarrow \infty} \frac{\ln \left(1 + \frac{k}{n}\right)}{1/n} \quad \text{has IF } \frac{0}{0} \therefore \text{we will use L'Hôpital's rule.}$$

$$\ln y = \lim_{n \rightarrow \infty} \frac{\frac{1}{1 + \frac{k}{n}} \cdot \left(-\frac{k}{n^2}\right)}{-1/n^2}$$

$$\ln y = \lim_{n \rightarrow \infty} \frac{\frac{-kn}{n^3 + kn^2}}{-1/n^2}$$

$$\ln y = \lim_{n \rightarrow \infty} \frac{kn^2}{n^3 + kn^2}$$

$$\ln y = k$$

$$y = e^k$$

Question 2. §9.2#40 (3 marks) Find the sum of the convergent series:

$$\sum_{n=0}^{\infty} 6 \left(\frac{4}{5}\right)^n$$

The following is a geometric series with $r = \frac{4}{5}$ since $|r| < 1$, it converges and its sum is

$$\begin{aligned} \sum_{n=0}^{\infty} 6 \left(\frac{4}{5}\right)^n &= \frac{a}{1-r} \\ &= \frac{6}{1 - \frac{4}{5}} = 30 \end{aligned}$$