

Quiz #4 since the first course by Yann

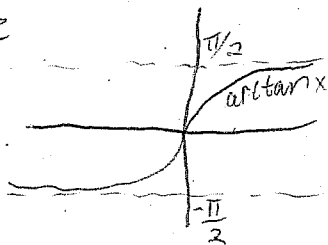
This quiz is graded out of 10 marks. No books, graphing calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §9.3#15 ¹⁰ marks ^{the} Use Integral Test to determine the convergence or divergence.

$$\sum_{n=1}^{\infty} \frac{\arctan n}{n^2 + 1}$$

Let $f(x) = \frac{\arctan x}{x^2 + 1}$ notice $f(x) > 0$ for $x \geq 1$

since



$$f'(x) = \frac{\left(\frac{1}{x^2+1}\right)(x^2+1) - (\arctan x)(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(1 - (\arctan x)(x^2+1))}{(x^2+1)^2} < 0 \text{ for } x \geq 1$$

since $\arctan(1) = \frac{\pi}{4}$ and $\arctan$ is increasing.

$$\int_1^{\infty} \frac{\arctan x}{x^2 + 1} dx = \lim_{b \rightarrow \infty} \int_1^b \frac{\arctan x}{1 + x^2} dx$$

Let $u = \arctan x$
 $du = \frac{dx}{1+x^2}$

$u(1) = \arctan 1 = \frac{\pi}{4}$
 $u(b) = \arctan b$

$$= \lim_{b \rightarrow \infty} \int_{\frac{\pi}{4}}^{\arctan b} u du$$

$$= \lim_{b \rightarrow \infty} \left[\frac{u^2}{2} \right]_{\frac{\pi}{4}}^{\arctan b}$$

$$= \lim_{b \rightarrow \infty} \frac{(\arctan b)^2}{2} - \frac{\left(\frac{\pi}{4}\right)^2}{2}$$

$$= \left(\frac{\pi}{2}\right)^2 / 2 - \frac{\pi^2 / 16}{2}$$

$$= \pi^2 / 4 / 2 - \frac{\pi^2}{32}$$

$$= \frac{\pi^2}{8} - \frac{\pi^2}{32} = \frac{3\pi^2}{32}$$

converges by the integral test.