Dawson College: Calculus II: 201-NYB-05-C1: Winter 2008

Name: YANN LAMONTAGNE
Student ID: _____

Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §4.6#9 *(10 marks)*
Use the Trapezoidal Rule and Simpson's Rule to approximate the value of the definite integral for $n = 4$. Round your answer to four decimal places and compare the results with the exact value of the definite integral.

$$\int_1^2 \frac{1}{(x+1)^2}\,dx$$

Definite integral using the fundamental theorem of calculus:

Let $u = x+1$, $du = dx$

$$\int_1^2 \frac{1}{u^2}dx = \int_2^3 \frac{1}{u^2}du = -u^{-1}\Big|_2^3 = -3^{-1} + 2^{-1} = \frac{1}{6} \doteq 0.1666$$

$\Delta x = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4}$ $\quad x_i = a + i\Delta x$, $\quad x_0 = 1 + 0(\frac{1}{4}) = 1$, $\quad x_1 = 1 + 1(\frac{1}{4}) = \frac{5}{4}$

$x_2 = 1 + 2(\frac{1}{4}) = \frac{6}{4}$, $\quad x_3 = 1 + 3(\frac{1}{4}) = \frac{7}{4}$

$x_4 = 1 + 4(\frac{1}{4}) = 2$

Trapezoidal Rule:

$$\int_1^2 f(x)dx \approx \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$$

$$= \frac{1}{8}\left[\frac{1}{(1+1)^2} + \frac{2}{(1+\frac{5}{4})^2} + \frac{2}{(1+\frac{6}{4})^2} + \frac{2}{(1+\frac{7}{4})^2} + \frac{1}{(1+2)^2}\right]$$

$$= \frac{1}{8}\left[\frac{1}{4} + \frac{32}{81} + \frac{32}{100} + \frac{32}{121} + \frac{1}{9}\right] \doteq 0.1676$$

Simpson's Rule:

$$\int_1^2 f(x)dx \approx \frac{b-a}{3n}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$$

$$\approx \frac{1}{12}\left[\frac{1}{4} + \frac{64}{81} + \frac{32}{100} + \frac{64}{100} + \frac{1}{9}\right] \doteq 0.1667$$

Simpson's rule is the best approx. for this function.