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Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §8.7#39 *(5 marks)*
Evaluate the following limit, using L'Hôpital's Rule if necessary:

$\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right)$ Indeterminate form $\infty\cdot 0$, therefore we rewrite the limit as $\lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{1/x}$ and its indeterminate form is $\frac{0}{0}$

$\therefore$, we use L'Hôpital's Rule.

$$\lim_{x\to\infty} \frac{\cos\left(\frac{1}{x}\right)\frac{d}{dx}\left[\frac{1}{x}\right]}{\frac{d}{dx}\left[\frac{1}{x}\right]} = \lim_{x\to\infty} \cos\left(\frac{1}{x}\right) = 1$$

$\therefore \lim_{x\to\infty} x\sin\left(\frac{1}{x}\right) = 1$

Question 2. §8.8# *(5 marks)*
Evaluate the following improper integral if it converges:

$$\int_4^\infty \frac{1}{x(\ln x)^3}\,dx = \lim_{b\to\infty}\int_4^b \frac{1}{x(\ln x)^3}\,dx$$

$$= \lim_{b\to\infty}\left[\frac{-1}{2(\ln x)^2}\right]\Bigg|_4^b$$

$$= \lim_{b\to\infty} \frac{-1}{2(\ln b)^2} + \lim_{b\to\infty} \frac{1}{2(\ln 4)^2}$$

$$= 0 + \frac{1}{2(\ln 4)^2} = \frac{1}{2(\ln 4)^2}$$