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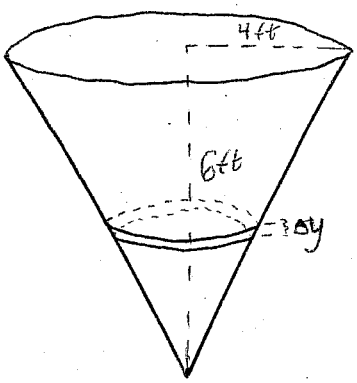
Quiz 9

This quiz is graded out of 10 marks. No books, graphing calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

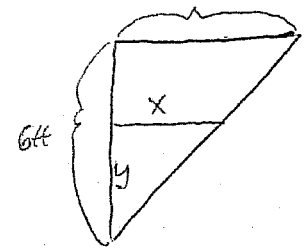
Question 1. §7.5#25 *(10 marks)*
An open tank has the shape of a circular cone with its tip oriented downward. The tank is 8 feet across the top and 6 feet high. How much work is done in emptying the tank by pumping the water over the top edge? (The water weighs 62.4 pounds per cubic feet)



Volume of slice:

$\Delta V = \pi x^2 \Delta y$

We want the volume of the slice be with respect to y only.



$\frac{x}{y} = \frac{4}{6}$

$x = \frac{4y}{6}$

$\therefore \Delta V = \pi \left(\frac{4y}{6}\right)^2 \Delta y$

$\Delta V = \pi \frac{16}{36} y^2 \Delta y$

The force excerted by slice:

$\Delta F = \Delta V \text{ (weigh per cubic feet)}$

$= \frac{4}{9} \pi y^2 (62.4) \Delta y$

$= \frac{416}{15} \pi y^2 \Delta y$

Work to move slice:

$\Delta W = \Delta F D$

$= \frac{416}{15} \pi y^2 \Delta y \, (6-y)$

$\therefore W = \int_0^6 \frac{416}{15} \pi y^2 (6-y) dy = \frac{416}{15} \pi \int_0^6 y^2(6-y) dy = \frac{416\pi}{15} \int_0^6 6y^2 - y^3 dy$

$= \frac{416\pi}{15} \left[2y^3 - \frac{y^4}{4} \right]_0^6$

$= \frac{416\pi}{15} \left[2(6)^3 - \frac{(6)^4}{4} \right]$

$= \frac{416\pi}{15} \left[432 - 324 \right]$

$= \frac{14976\pi}{15} \text{ ft} \cdot \text{lb} \doteq 9410 \text{ ft}$