$$\sum_{n=1}^{\infty} \frac{1}{9n^2+3n-2} = \sum_{n=1}^{\infty} \frac{1}{(3n-1)(3n+2)}$$

Lets decompose using partial fractions

$$\frac{1}{(3n-1)(3n+2)} = \frac{A}{(3n-1)} + \frac{B}{(3n+2)}$$

$$1 = A(3n+2) + B(3n-1)$$

Lets solve for A, B using special values. (i.e $n = \frac{-2}{3}, \frac{1}{3}$

Let $n = \frac{-2}{3}$

$$1 = A(3(\tfrac{-2}{3})+2) + B(3(\tfrac{-2}{3})-1)$$

$$1 = A(0) \quad B$$

$$-\tfrac{1}{3} = B$$

Let $n = \frac{1}{3}$

$$1 = A(3(\tfrac{1}{3})+2) + B(3(\tfrac{1}{3})-1)$$

$$1 = A \cdot 3 + B(0)$$

$$\tfrac{1}{3} = A$$

$$\therefore \sum_{n=1}^{\infty} \frac{1}{9n^2+3n-2} = \sum_{n=1}^{\infty} \left[\frac{1/3}{3n-1} - \frac{1/3}{3n+2} \right]$$

$$= \frac{1}{3} \sum_{n=1}^{\infty} \left[\frac{1}{3n-1} - \frac{1}{3n+2} \right]$$

Lets look at the partial sum to find the sum.

$$S_n = a_1 + a_2 + a_3 + \ldots + a_{n-2} + a_{n-1} + a_n$$

$$S_n = \left[\frac{1}{2} - \cancel{\frac{1}{5}}\right] + \left[\cancel{\frac{1}{5}} - \cancel{\frac{1}{8}}\right] + \left[\cancel{\frac{1}{8}} - \frac{1}{11}\right] + \cdots + \left[\frac{1}{3(n-2)-1} - \cancel{\frac{1}{3(n-2)+2}}\right]$$

$$+ \left[\cancel{\frac{1}{3(n-1)-1}} - \cancel{\frac{1}{3(n-1)+2}}\right] + \left[\cancel{\frac{1}{3n-1}} - \frac{1}{3n+2}\right]$$

$$= \frac{1}{2} - \frac{1}{3n+2}$$

Continue

$$= \frac{1}{3} \lim_{n\to\infty} S_n$$

$$= \frac{1}{3}\left[\lim_{n\to\infty}\left(\frac{1}{2} - \cancelto{0}{\frac{1}{3n+2}}\right)\right]$$

$$= \frac{1}{3}\left[\frac{1}{2}\right]$$

$$= \frac{1}{6}$$