Dawson College: **Calculus II**: 201-NYB-05-C1: Winter 2008

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Test 1

This test is graded out of 40 marks. No books, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formula you might need:

$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin \frac{u}{a} + C \qquad \int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan \frac{u}{a} + C$

$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec} \frac{|u|}{a} + C$

Question 1. *(2 mark)*
Integrate the following indefinite integral:

$$\int (3\sin x + 5\cos x)\, dx = 3\int \sin x\, dx + 5\int \cos x\, dx$$

$$= -3\cos x + 5\sin x + C$$

Question 2. *(5 marks)*
Integrate the following indefinite integral:

$$\int \frac{1}{\theta^2}\cos\frac{1}{\theta}\, d\theta \overset{①}{=} \int \frac{1}{\theta^2}\cos u\, d\theta$$

Let ① $u = \frac{1}{\theta}$

$$\overset{②}{=} -\int \cos u\, du$$

then $du = \frac{-d\theta}{\theta^2}$

② $-du = \frac{d\theta}{\theta^2}$

$$= -\sin u + C$$

$$= -\sin\frac{1}{\theta} + C$$

Question 3. *(5 marks)*
Integrate the following indefinite integral:

$$\int \frac{(\ln x)^2}{x}\,dx$$

Let ① $u = \ln x$ then

② $du = \frac{dx}{x}$

$$\overset{①}{=} \int \frac{(u)^2}{x}\,dx$$

$$\overset{②}{=} \int u^2\,du$$

$$= \frac{u^3}{3} + C$$

$$\overset{①}{=} \frac{(\ln x)^3}{3} + C$$

Question 4. *(5 marks)*
Integrate the following indfinite integral *(hint: use inverse trigonometric funtion)*:

$$\int \frac{3}{2\sqrt{x}(4+x)}\,dx$$

Let ① $u = \sqrt{x}$ then

② $du = \frac{dx}{2\sqrt{x}}$

$$= \int \frac{3}{2\sqrt{x}\,(2^2 + (\sqrt{x})^2)}\,dx$$

$$\overset{①}{=} \int \frac{3}{2\sqrt{x}\,(2^2 + u^2)}\,dx$$

$$\overset{②}{=} 3\int \frac{du}{(2^2+u^2)}$$

$$= 3\left[\frac{1}{2}\arctan\frac{u}{2} + C\right]$$

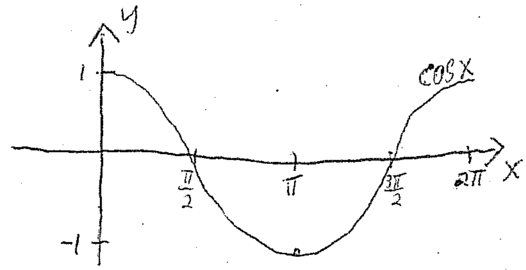
$$= \frac{3}{2}\arctan\frac{\sqrt{x}}{2} + C$$

Question 5. *(3 marks)*
Given $\int_a^b f(x)\,dx = 3$, $\int_a^b g(x)\,dx = 3$ and $\int_b^c f(x)\,dx = 4$ evaluate the following definite integrals:

1. $$\int_c^a f(x)\,dx = -\left[\int_a^c f(x)\,dx\right] = -\left[\int_a^b f(x)\,dx + \int_b^c f(x)\,dx\right] = -\left[3+4\right] = -7$$

2. $$\int_b^a f(x)+g(x)\,dx = -\left[\int_a^b f(x)+g(x)\,dx\right] = -\left[\int_a^b f(x)\,dx + \int_a^b g(x)\,dx\right] = -\left[3+3\right] = -6$$

3. $$\int_a^b g(x)\,dx = 3$$



Question 6. *(5 marks)*
Evaluate the following definite integral:

$$\int_0^{\pi/2} \cos x \sin x \, dx$$

Let $u = \cos x$ ①

then $du = -\sin x \, dx$ ②

$$\overset{①}{=} \int_0^{\pi/2} u \sin x \, dx$$

$$\overset{②}{=} -\int_{u(0)}^{u(\frac{\pi}{2})} u \, du$$

$$= -\int_1^0 u \, du$$

$$= -\left[\frac{u^2}{2} \Big|_1^0 \right]$$

$$= -\left[\frac{0^2}{2} - \frac{1^2}{2} \right] = \frac{1}{2}$$

Question 7. *(3 marks)*
Use the Second Fundamental Theorem of Calculus to find $F'(x)$.

$$F(x) = \int_0^{x^2} \cos y^2 \, dy$$

Let ① $u = x^2$

$$\frac{d}{dx}\left[F(x)\right] = \frac{d}{du}\left[\int_0^u \cos y^2 dy\right] \frac{d}{dx}[u]$$

$$= (\cos u^2)(2x)$$

$$\overset{①}{=} \cos (x^2)^2 (2x)$$

$$= (\cos x^4)(2x)$$

$$= 2x \cos x^4$$

Question 8. *(5 marks)*
Evaluate the following definite integral:

$$\int_{-1}^{2} x(x^2+1)^3 \, dx$$

Let ① $u = x^2 + 1$

then $\frac{du}{2} = x \, dx$

$$\overset{①}{=} \int_{-1}^{2} x(u)^3 dx$$

$$\overset{②}{=} \frac{1}{2}\int_{u(-1)}^{u(2)} u^3 du$$

$$= \frac{1}{2}\int_2^5 u^3 du$$

$$= \frac{1}{2}\left[\frac{u^4}{4} \Big|_2^5 \right]$$

$$= \frac{1}{2}\left[\frac{(5)^4}{4} - \frac{2^4}{4} \right]$$

$$= \frac{1}{2}\left[\frac{625}{4} - \frac{16}{4} \right]$$

$$= \frac{609}{8}$$

Question 9. *(4 marks)*
Evaluate the following indefinite integral:

$$\int e^{3-x}\,dx \overset{①}{=} \int e^{u}\,dx$$

Let ① $u = 3-x$ then

$du = u'(x)\,dx$

② $du = -dx$

$$\overset{②}{=} -\int e^{u}\,du$$

$$= -e^{u} + C$$

$$= -e^{3-x} + C$$

Question 10. *(3 marks)*
Find the average value of the function $f(x) = 4x^2$ over the interval $[2,4]$:

$$\text{Avg value of function } f = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{4-2}\int_2^4 4x^2\,dx$$

$$= \frac{1}{2}\left[\frac{4x^3}{3}\Big|_2^4\right]$$

$$= \frac{4}{6}\left[4^3 - 2^3\right] = \frac{112}{3}$$

Bonus Question. *(3 marks)*
Integrate the following indefinite intergral:

$$\int \frac{\ln\left(\frac{e}{x^2}\right)}{x}\,dx = \int \frac{\ln e - \ln x^2}{x}\,dx$$

$$= \int \frac{1}{x}\,dx - 2\int \frac{\ln x}{x}\,dx$$

$$= \ln|x| - (\ln x)^2 + C$$