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# Quiz 4

**Question 1.** *(5 marks)*

**(a)** Write the following system of equations as a matrix equation ($Ax = b$).

$$\begin{array}{rcrcrcl} x_1 & + & 3x_2 & +x_3 & = & 4 \\ 2x_1 & + & 2x_2 & + & x_3 & = & -1 \\ 2x_1 & + & 3x_2 & + & x_3 & = & 3 \end{array}$$

$$\underset{A}{\begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}} \underset{x}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} = \underset{b}{\begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix}}$$

**(b)** Use the following information to solve the above system:

$$A^{-1} = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 2 & 1 \\ 2 & 3 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix}$$

$$Ax = b \Rightarrow x = A^{-1}b$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 2 & 3 & -4 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -7 \end{bmatrix}$$

$$(x_1, x_2, x_3) = (-1, 4, -7)$$

**Question 2.** *(2 marks)* Compute $A^{-2}$ by inspection:

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1/9 \end{bmatrix}$$

**Question 3.** *(3 marks)* Circle the lower triangular matrices.

$$A = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 7 & 0 & 0 \\ 5 & 3 & -4 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & -4 \end{bmatrix} \quad D = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 3 & 7 & 0 & 0 \\ 9 & 6 & 7 & 0 \\ 0 & 0 & 5 & 3 \end{bmatrix}$$