March 4, 2009

Name: SOLUTIONS

Student ID: _____

Quiz 5

Question 1. *(6 marks)* Find the determinant of A using the expansion definition (not row/column operations).

$$A = \begin{bmatrix} 2 & 0 & 3 & -1 \\ 1 & 0 & 0 & 1 \\ 5 & 2 & 5 & -3 \\ 2 & 0 & 4 & 0 \end{bmatrix}$$

ALONG COLUMN 2

$$\det A = 0 + 0 - 2\det\begin{bmatrix} 2 & 3 & -1 \\ 1 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix} + 0$$

ALONG ROW 2

$$= -2\left(-1\det\begin{bmatrix} 3 & -1 \\ 4 & 0 \end{bmatrix} + 0 - 1\det\begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix}\right)$$

$$= -2\left(-(0+4) - (8-6)\right) = -2(-4-2)$$

$$= 12$$

Question 2. *(4 marks)* Solve the following system using Cramer's Rule.

$$\begin{array}{rcrcrcl} x_1 & & & + & x_3 & = & 2 \\ 2x_1 & - & x_2 & + & 5x_3 & = & -2 \\ -x_1 & + & 3x_2 & & & = & 7 \end{array}$$

Given that:

$$\det\begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 5 \\ -1 & 3 & 0 \end{bmatrix} = -10, \quad \det\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 5 \\ 7 & 3 & 0 \end{bmatrix} = -29,$$

$$\det\begin{bmatrix} 1 & 2 & 1 \\ 2 & -2 & 5 \\ -1 & 7 & 0 \end{bmatrix} = -33, \quad \det\begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & -2 \\ -1 & 3 & 7 \end{bmatrix} = 9,$$

$$x_1 = \frac{\det A_1}{\det A} = \frac{-29}{-10} = \frac{29}{10} \qquad x_2 = \frac{\det A_2}{\det A} = \frac{-33}{-10} = \frac{33}{10}$$

$$x_3 = \frac{\det A_3}{\det A} = \frac{9}{-10} = -\frac{9}{10} \quad \therefore \quad (x_1, x_2, x_3) = \left(\frac{29}{10}, \frac{33}{10}, -\frac{9}{10}\right)$$