QUIZ / ASSIGNMENT #5 SOLUTIONS

1) $\int_0^{\pi/2} \sec^4(t/2)\,dt$

$= 2\int_0^{\pi/4} \sec^4(u)\,du$

$= 2\int_0^{\frac{\pi}{4}} \sec^2 u \sec^2 u\,du$

$= 2\int_0^{\frac{\pi}{4}} (\tan^2 u + 1)\sec^2 u\,du$

$= 2\int_0^{1} (v^2+1)\,dv$

$= 2\left[\frac{v^3}{3} + v\right]_0^1$

$= 2\left[\left(\frac{1}{3}+1\right) - \left(\frac{0}{3}+0\right)\right]$

$= 2\left(\frac{4}{3}\right) = \frac{8}{3}$

LET $u = \frac{t}{2}$

$du = \frac{1}{2}dt \Rightarrow dt = 2du$

IF $t=0 \Rightarrow u=0$

$t = \frac{\pi}{2} \Rightarrow u = \frac{\pi}{4}$

LET $v = \tan u$

$dv = \sec^2 u\,du$

IF $u=0 \Rightarrow v = \tan 0 = 0$

$u = \frac{\pi}{4} \Rightarrow v = \tan\frac{\pi}{4} = 1$

2) $\int \csc^4 x \cot^6 x \, dx$

$= \int \csc^2 x \csc^2 x \cot^6 x \, dx$

$= \int \csc^2 x (1 + \cot^2 x) \cot^6 x \, dx$

LET $u = \cot x$

$du = -\csc^2 x \, dx$

$dx = \frac{du}{-\csc^2 x}$

$= \int \csc^2 x (1 + u^2) u^6 \frac{du}{-\csc^2 x}$

$= -\int (1 + u^2) u^6 \, du$

$= -\int (u^6 + u^8) \, du$

$= -\left(\frac{u^7}{7} + \frac{u^9}{9} \right) + C$

$= -\frac{1}{7} \cot^7 x - \frac{1}{9} \cot^9 x + C$

3) $\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16-x^2}}\,dx$

LET $x = 4\sin\theta \quad -\pi/2 \le \theta \le \pi/2$

$dx = 4\cos\theta\,d\theta$

IF $x=0 \Rightarrow \theta = 0$

$x = 2\sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$

AND $\sqrt{16-x^2} = \sqrt{16-(4\sin\theta)^2}$

$= \sqrt{16(1-\sin^2\theta)} = 4\sqrt{\cos^2\theta}$

$= 4|\cos\theta| = 4\cos\theta$

SINCE $\cos\theta \ge 0$ ON $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

$= \int_0^{\pi/3} \frac{(4\sin\theta)^3\, 4\cos\theta\,d\theta}{4\cos\theta}$

$= 64\int_0^{\pi/3} \sin^3\theta\,d\theta$

$= 64\int_0^{\pi/3} \sin^2\theta\sin\theta\,d\theta$

$= 64\int_0^{\pi/3} (1-\cos^2\theta)\sin\theta\,d\theta$

LET $u = \cos\theta$

$du = -\sin\theta$

IF $\theta = 0 \Rightarrow u = \cos 0 = 1$

$\theta = \frac{\pi}{3} \Rightarrow u = \cos\frac{\pi}{3} = \frac{1}{2}$

$= -64\int_1^{1/2} (1-u^2)\,du$

$= -64\left[u - \frac{u^3}{3}\right]_1^{1/2}$

$= -64\left[\left(\frac{1}{2} - \frac{(1/2)^3}{3}\right) - \left(1 - \frac{1}{3}\right)\right]$

$= -64\left[\frac{1}{2} - \frac{1}{24} - \frac{2}{3}\right]$

$= -64\left(\frac{-5}{24}\right)$

$= 40/3$

4) $\int \frac{dx}{x^2\sqrt{16x^2-9}}$

LET $u = 4x$

$du = 4dx$

$= \int \frac{1}{\left(\frac{u}{4}\right)^2\sqrt{u^2-9}} \cdot \frac{du}{4}$

$= 4\int \frac{1}{u^2\sqrt{u^2-9}}\,du$

LET $u = 3\sec\theta$ ON $0 \le \theta \le \frac{\pi}{2}$ OR $\pi \le \theta \le \frac{3\pi}{2}$

$du = 3\sec\theta\tan\theta\,d\theta$

AND

$\sqrt{u^2-9} = \sqrt{9\sec^2\theta - 9}$

$= 3\sqrt{\sec^2\theta - 1} = 3\sqrt{\tan^2\theta}$

$= 3|\tan\theta| = 3\tan\theta$

SINCE $\tan\theta \ge 0$ ON

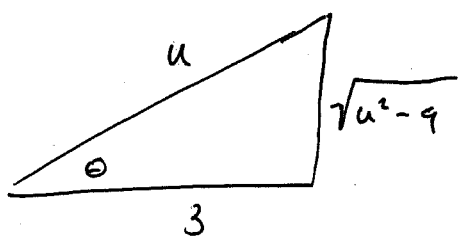
$0 \le \theta \le \frac{\pi}{2}$ OR $\pi \le \theta \le \frac{3\pi}{2}$

$= 4\int \frac{3\sec\theta\tan\theta\,d\theta}{(3\sec\theta)^2 \cdot 3\tan\theta}$

$= \frac{4}{9}\int \frac{d\theta}{\sec\theta}$

$= \frac{4}{9}\int \cos\theta\,d\theta$

$= \frac{4}{9}\sin\theta + C$



$= \frac{4}{9}\cdot\frac{\sqrt{u^2-9}}{u} + C$

$= \frac{4}{9}\cdot\frac{\sqrt{(4x)^2-9}}{4x} + C$

$= \frac{\sqrt{16x^2-9}}{9x} + C$

5) $\int \frac{dt}{\sqrt{t^2-6t+13}}$

$t^2-6t+13 = (t^2-6t+9)+13-9$

$= (t-3)^2+4$

LET $u = t-3$

$du = dt$

$= \int \frac{dt}{\sqrt{(t-3)^2+4}}$

$= \int \frac{du}{\sqrt{u^2+4}}$

LET $u = 2\tan\theta$ ON $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

$du = 2\sec^2\theta\, d\theta$

$\sqrt{4\tan^2\theta+4} = \sqrt{4(\tan^2\theta+1)}$

$= 2\sqrt{\tan^2\theta+1} = 2\sqrt{\sec^2\theta}$

$= 2|\sec\theta| = 2\sec\theta$

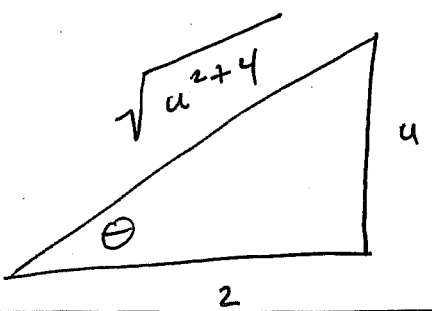
SINCE $\sec\theta > 0$ ON $-\frac{\pi}{2} < \theta \leq \pi/2$

$= \int \frac{2\sec^2\theta\, d\theta}{2\sec\theta}$

$= \int \sec\theta\, d\theta$

$= \ln|\sec\theta+\tan\theta| + C$

$u = 2\tan\theta \Rightarrow \tan\theta = \frac{u}{2}$



$= \ln\left|\frac{\sqrt{u^2+4}}{2} + \frac{u}{2}\right| + C$

$= \ln\left|\frac{\sqrt{(t-3)^2+4}}{2} + \frac{t-3}{2}\right| + C$