ASSIGNMENT # 11 SOLUTIONS

SECTION 7.4

#2 $y = \sqrt{4-x^2} \Rightarrow \frac{dy}{dx} = -\frac{x}{\sqrt{4-x^2}}$

$$\therefore L = \int_0^2 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_0^2 \sqrt{1+\left(-\frac{x}{\sqrt{4-x^2}}\right)^2}\,dx$$

$$= \int_0^2 \sqrt{1+\frac{x^2}{4-x^2}}\,dx = \int_0^2 \sqrt{\frac{4-x^2+x^2}{4-x^2}}\,dx$$

$$= \underbrace{\int_0^2 \frac{2}{\sqrt{4-x^2}}\,dx}_{\text{IMPROPER INTEGRAL}} = \lim_{t\to 2^-}\int_0^t \frac{2}{\sqrt{4-x^2}}\,dx$$

LET $u = \frac{x}{2}$

$du = \frac{1}{2}dx$

$2\,du = dx$

$x=0 \Rightarrow u=0$

$x=2 \Rightarrow u=1$

LET'S EVALUATE:

$$\int \frac{2}{\sqrt{4-x^2}}\,dx = \int \frac{\cancel{2}}{\cancel{2}\sqrt{1-\left(\frac{x}{2}\right)^2}}\,dx = \int \frac{2\,du}{\sqrt{1-u^2}}$$

$$= 2\arcsin u + C = 2\arcsin\left(\frac{x}{2}\right) + C$$

$$\therefore L = \lim_{t\to 2^-}\left[2\arcsin\left(\frac{x}{2}\right)\right]_0^t$$

$$= \lim_{t\to 2^-}\left[2\arcsin\left(\frac{t}{2}\right) - 2\arcsin(0)\right]$$

$$= \left[2\left(\frac{\pi}{2}\right) - 2(0)\right] = \pi \text{ units}$$

#6 $y = \frac{x^2}{2} - \frac{\ln x}{4} \Rightarrow \frac{dy}{dx} = x - \frac{1}{4x}$

$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \left(x^2 - \frac{1}{2} + \frac{1}{16x^2}\right) = x^2 + \frac{1}{2} + \frac{1}{16x^2}$

$= \left(x + \frac{1}{4x}\right)^2$

$\therefore L = \int_2^4 \sqrt{\left(x + \frac{1}{4x}\right)^2}\,dx = \left[\frac{x^2}{2} + \frac{1}{4}\ln x\right]_2^4$

$= \left(8 + \frac{\ln 4}{4}\right) - \left(2 + \frac{\ln 2}{4}\right)$

$= 6 + \frac{\ln 4}{4} - \frac{\ln 2}{4}$ units

#8, $y = \ln(\cos x) \Rightarrow \frac{dy}{dx} = -\tan x$

$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \tan^2 x = \sec^2 x$ so

$L = \int_0^{\frac{\pi}{3}} \sqrt{\sec^2 x}\,dx = \int_0^{\frac{\pi}{3}} |\sec x|\,dx = \int_0^{\pi/3} \sec x\,dx$

$\sec x \geq 0$ on $0 \leq x \leq \pi/4$

$= \left[\ln(\sec x + \tan x)\right]_0^{\frac{\pi}{3}} = \ln(\sqrt{2} + \sqrt{3}) - \ln(1 + 0)$

$= \ln(\sqrt{2} + \sqrt{3})$ units

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SECTION 8.1

#8 $a_n = 3 + (-1)^{n+1} \cdot 2$

#15 $a_n = \dfrac{(-1)^{n-1} \cdot n}{n^2+1}$

$\therefore \lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \left| \dfrac{(-1)^{n-1} \cdot n}{n^2+1} \right| = \lim_{n\to\infty} \dfrac{n}{n^2+1}$

$= \lim_{n\to\infty} \dfrac{1/n}{1+1/n^2} = \dfrac{0}{1+0} = 0$

$\therefore \lim_{n\to\infty} a_n = \lim_{n\to\infty} \dfrac{(-1)^{n-1} \cdot n}{n^2+1} = 0$ (CONVERGES)

#16 $a_n = \dfrac{(-1)^n n^3}{n^3+2n^2+1}$

NOW $\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \left| \dfrac{(-1)^n n^3}{n^3+2n^2+1} \right| = \lim_{n\to\infty} \dfrac{n^3}{n^3+2n^2+1}$

$= \lim_{n\to\infty} \dfrac{1}{1+2/n+1/n^3} = 1$

BUT THE TERMS OF $\{a_n\}$ ALTERNATE IN SIGN

SO $a_1, a_3, a_5, \ldots$ CONVERGES TO -1 AND

$a_2, a_4, a_6, \ldots$ CONVERGES TO 1

$\therefore$ THE SEQUENCE DIVERGES.

#26 $a_n = \frac{(\ln n)^2}{n}$

Now $\lim_{x\to\infty} \frac{(\ln x)^2}{x} = \text{"}\frac{\infty}{\infty}\text{"} \overset{H}{=} \lim_{n\to\infty} \frac{2\ln x \cdot \frac{1}{x}}{1}$

$= \lim_{n\to\infty} \frac{2\ln x}{x} = \text{"}\frac{\infty}{\infty}\text{"} \overset{H}{=} \lim_{n\to\infty} \frac{2\cdot\frac{1}{x}}{1} = 0$

$\therefore \lim_{n\to\infty} \frac{(\ln n)^2}{n} = 0$ (CONVERGES)