Dawson College: **Calculus II**: 201-NYB-05 12 Due April 19, 2010

Last Name: ____________________

First Name: ____________________

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Quiz/Assignment 9

Question 1. *(5 marks)* Evaluate the following limit:

$$\lim_{x\to 0^+} (\cos x)^{1/x^2}$$

Question 2. Evaluate the following:

(a) *(5 marks)*

$$\int_0^1 \frac{dx}{\sqrt{1-x^2}}$$

(b) *(5 marks)*

$$\int_{-1}^1 \frac{e^x}{e^x-1}\,dx$$

(c) *(5 marks)*

$$\int_0^2 z^2 \ln z \, dz$$

Question 3.

(a) *(5 marks)* Find the area bound by the curves $y = \sin x$ and $y = e^x$ and the lines $x = 0$ and $x = \pi/2$.

(b) *(5 marks)* Find the area bounded by the curves $x = 2y^2$ and $x = 4 + y^2$.

1) $\lim_{x\to 0^+} (\cos x)^{1/x^2} = "1^\infty"$

LET $y = (\cos x)^{1/x^2}$

$\ln y = \frac{1}{x^2} \ln(\cos x)$

$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{1}{x^2} \ln(\cos x) = "\infty \cdot 0"$

$= \lim_{x\to 0^+} \frac{\ln(\cos x)}{x^2} = "\frac{0}{0}" \overset{(H)}{=} \lim_{x\to 0^+} \frac{\frac{1}{\cos x}(-\sin x)}{2x}$

$= \lim_{x\to 0^+} \frac{-\tan x}{2x} = "\frac{0}{0}" \overset{(H)}{=} \lim_{x\to 0^+} \frac{-\sec^2 x}{2} = \frac{-1}{2}$

$\therefore \lim_{x\to 0^+} (\cos x)^{1/x^2} = \lim_{x\to 0^+} y = \lim_{x\to 0^+} e^{\ln y}$

$= e^{\lim_{x\to 0^+} \ln y} = e^{-1/2}$

NOT ASSIGNED

$\int_{-\infty}^{\infty} x^2 e^{-x^3} dx = \underbrace{\int_{-\infty}^{0} x^2 e^{-x^3} dx}_{I_1} + \underbrace{\int_{0}^{\infty} x^2 e^{-x^3} dx}_{I_2}$

$\int x^2 e^{-x^3} dx = \int x^2 e^u \frac{du}{-3x^2} = -\frac{1}{3}\int e^u du$

LET $u = -x^3$
$du = -3x^2 dx$
$dx = \frac{du}{-3x^2}$

$= -\frac{1}{3} e^u + C = -\frac{1}{3} e^{-x^3} + C$

$$\therefore\ I_1 = \int_{-\infty}^{0} x^2 e^{-x^3}\,dx = \lim_{t\to-\infty} \int_t^0 x^2 e^{-x^3}\,dx$$

$$= \lim_{t\to-\infty} \left[-\frac{1}{3}e^{-x^3}\right]_t^0 = \lim_{t\to-\infty}\left[-\frac{1}{3}e^{-(0)^3} + \frac{1}{3}e^{-t^3}\right]$$

$$= \infty$$

$\therefore$ THE INTEGRAL DIVERGES.

2a) $$\int_0^1 \frac{dx}{\sqrt{1-x^2}} = \lim_{t\to1^-} \int_0^t \frac{dx}{\sqrt{1-x^2}} = \lim_{t\to1^-} \left[\arcsin x\right]_0^t$$

$$= \lim_{t\to1^-} \left[\arcsin t - \arcsin 0\right]$$

$$= \lim_{t\to1^-} \arcsin t = \arcsin(1) = \frac{\pi}{2}$$

b) $$\int_{-1}^{1} \frac{e^x}{e^x-1}\,dx = \underbrace{\int_{-1}^{0} \frac{e^x}{e^x-1}\,dx}_{I_1} + \underbrace{\int_0^1 \frac{e^x}{e^x-1}\,dx}_{I_2}$$

$$\int \frac{e^x}{e^x-1}\,dx = \int \frac{1}{u}\,du$$

$$= \ln|u| + C = \ln|e^x-1| + C$$

LET $u = e^x - 1\,dx$

$du = e^x\,dx$

$$I_1 = \lim_{t \to 0^-} \int_{-1}^{t} \frac{e^x}{e^x - 1}\, dx = \lim_{t \to 0^-} \Big[\ln|e^x - 1| \Big]_{-1}^{t}$$

$$= \lim_{t \to 0^-} \Big[\ln|e^t - 1| - \ln|e^{-1} - 1| \Big]$$

$$= -\infty$$

$\therefore$ THE INTEGRAL DIVERGES.

$$2c \quad I = \int_0^2 z^2 \ln z\, dz = \lim_{t \to 0^+} \int_t^2 z^2 \ln z\, dz$$

$$\int z^2 \ln z\, dz$$

LET
$u = \ln z \qquad dv = z^2\, dz$
$du = \frac{1}{z}\, dz \qquad v = \frac{z^3}{3}$

$$= \frac{z^3}{3} \ln z - \frac{1}{3} \int z^2\, dz$$

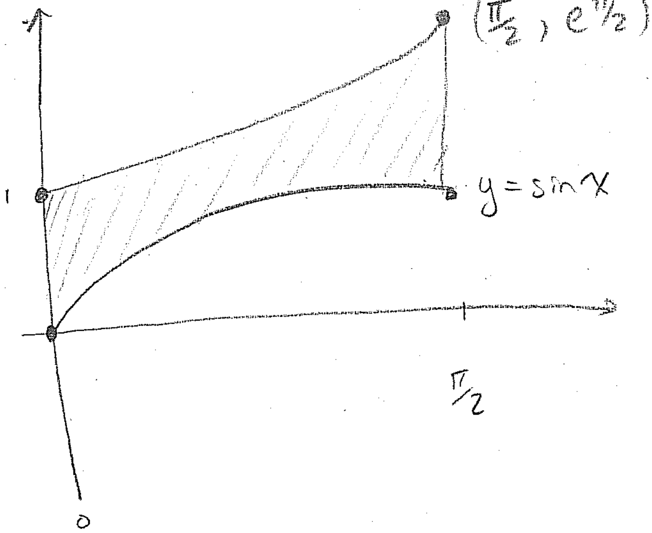
$$= \frac{z^3}{3} \ln z - \frac{1}{9} z^3 + C$$

$$\therefore I = \lim_{t \to 0^+} \Big[\frac{z^3}{3} \ln z - \frac{1}{9} z^3 \Big]_t^2$$

$$= \lim_{t \to 0^+} \Big[\frac{t^3}{3} \ln t - \frac{1}{9} t^3 - \frac{2^3}{3} + \frac{1}{9} (2)^2 \Big]$$

$$= -\infty \quad \therefore \text{ THE INTEGRAL DIVERGES}$$

3a)



$\therefore A = \int_0^{\pi/2} (e^x - \sin x)\,dx = \left[e^x + \cos x\right]_0^{\pi/2}$

$= \left(e^{\pi/2} + \cos\frac{\pi}{2}\right) - \left(e^0 + \cos 0\right) = e^{\pi/2} + 0 - 1 - 1$

$= e^{\pi/2} - 2$ units^2

3b) $x = 2y^2$ | $x = 4 + y^2$

IF $y = 0 \Rightarrow x = 0$ | IF $y = 0 \Rightarrow x = 4$

$(0, 0)$ | $(4, 0)$

INTERSECTION:

$2y^2 = 4 + y^2$

$y^2 - 4 = 0$

$(y+2)(y-2) = 0$

$y = -2, 2$

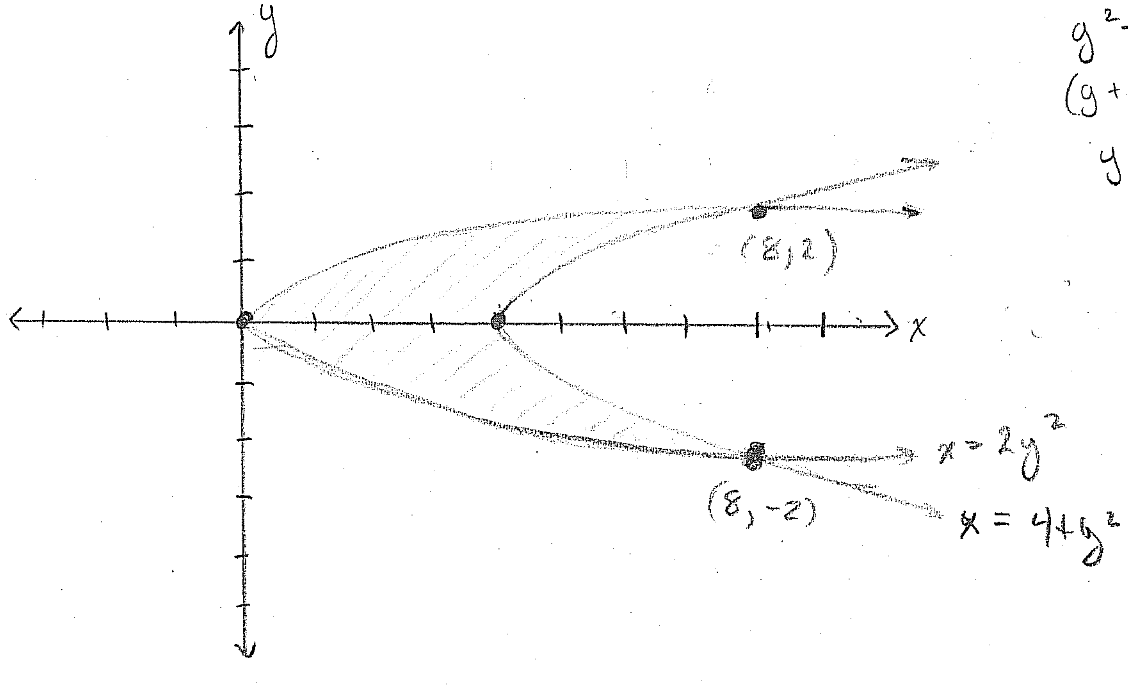
POINTS.

$x = 2(-2)^2 = 8$

$\therefore (8, -2)$

$x = 2(2)^2 = 8$

$(8, 2)$



$$\therefore A = \int_{-2}^{2} (x_2 - x_4)\,dy = \int_{-2}^{2} (4+y^2) - (2y^2)\,dy$$

$$= \int_{-2}^{2} (4 - y^2)\,dy = \left[4y - \frac{y^3}{3}\right]_{-2}^{2}$$

$$= \left[4(2) - \frac{(2)^3}{3}\right] - \left[4(-2) - \frac{(-2)^3}{3}\right]$$

$$= 8 - \frac{8}{3} + 8 - \frac{8}{3} = 16 - \frac{16}{3} = \frac{32}{3} \text{ units}^2$$