

Last Name: SOLUTIONS

First Name: _____

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Quiz 2

Question 1. (10 marks) Evaluate the following definite integrals using the definition of the integral:

$$\int_1^5 (x - 4x^2) dx$$

$$\bullet \Delta x = \frac{b-a}{n} = \frac{5-1}{n} = \frac{4}{n} \quad \bullet x_i = a + i\Delta x = 1 + \frac{4i}{n}$$

$$\bullet f(x_i) = x_i - 4(x_i)^2 = \left(1 + \frac{4i}{n}\right) - 4\left(1 + \frac{4i}{n}\right)^2 = 1 + \frac{4i}{n} - 4\left(1 + \frac{8i}{n} + \frac{16i^2}{n^2}\right)$$

$$= 1 + \frac{4i}{n} - 4 - \frac{32i}{n} - \frac{64i^2}{n^2} = -3 - \frac{28i}{n} - \frac{64i^2}{n^2}$$

$$\bullet f(x_i)\Delta x = \left[-3 - \frac{28i}{n} - \frac{64i^2}{n^2}\right]\left(\frac{4}{n}\right) = -\frac{12}{n} - \frac{112i}{n^2} - \frac{256i^2}{n^3}$$

$$\int_1^5 (x - 4x^2) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i)\Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(-\frac{12}{n} - \frac{112i}{n^2} - \frac{256i^2}{n^3}\right)$$

$$= \lim_{n \rightarrow \infty} \left(-\frac{12}{n} \sum_{i=1}^n 1 - \frac{112}{n^2} \sum_{i=1}^n i - \frac{256}{n^3} \sum_{i=1}^n i^2\right)$$

$$= \lim_{n \rightarrow \infty} \left[-\frac{12}{n} \cdot n - \frac{112}{n^2} \cdot \frac{n(n+1)}{2} - \frac{256}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n \rightarrow \infty} \left[-12 - \frac{112}{2} \cdot \frac{n}{n} \cdot \frac{n+1}{n} - \frac{256}{6} \cdot \frac{n}{n} \cdot \frac{n+1}{n} \cdot \frac{2n+1}{n}\right]$$

$$= \lim_{n \rightarrow \infty} \left[-12 - 56\left(1 + \frac{1}{n}\right) - \frac{128}{3} \cdot \left(1 + \frac{1}{n}\right)\left(2 + \frac{1}{n}\right)\right]$$

$$= -12 - 56 - \frac{128}{3} \cdot 2$$

$$= -\frac{460}{3}$$