Dawson College: **Calculus II**: 201-NYB-05-S02: Winter 2010

Name: _____
Student ID: _____

Test 1

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formulae:

$\sum_{i=1}^{n} c = cn$ where c is a constant $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Question 1. *(5 marks)* Evaluate using the definition of the definite integral

$$\int_1^2 3x^2 - 2x + 1 \, dx.$$

Question 2. *(5 marks)* Evaluate the definite integral:

$$\int_1^2 \sqrt{x}(x-1)(x-1) - \frac{1}{x}\, dx$$

Question 3. *(5 marks)* Evaluate the indefinite integral:

$$\int \frac{y^2}{\sqrt{y-1}}\, dy$$

Question 4. *(5 marks)* Find the average value of the function

$$f(x) = \frac{x^2}{x^2+3}$$

on the interval $[0, \sqrt{3}]$

Question 5. *(5 marks + 1 bonus mark to simplify completely)* Evaluate the expression:

$$\frac{d}{dx}\left[\int_{-\cos x}^{\cos x} t^2 \arctan t \, dt\right]$$

Question 6. *(5 marks)* Suppose $f(x)$ is continous over the real numbers and

$$\int_0^1 f(x)\,dx = 121.$$

Evaluate

$$\int_{-\pi}^{0} \sin\left(\frac{x}{2}\right) f\left(\cos\left(\frac{x}{2}\right)\right)\,dx.$$

Question 7. *(5 marks)* Evaluate the indefinite integral:

$$\int x \operatorname{arcsec} x\,dx$$

Question 8. *(5 marks)* Evaluate the definite integral:

$$\int_{\pi/18}^{\pi/12} t \csc 3t \cot 3t \; dt$$

Question 9. *(5 marks)* Prove: If $f(x)$ is a continuous and even function then

$$\int_{-a}^{a} f(x)\, dx = 2\int_{0}^{a} f(x)\, dx.$$

Bonus Question. *(3 marks)*
If $f(x)$ is a continuous function on a certain domain and satisfies

$$0 = \int_{98}^{x} f(t)\,dt - \arcsin(\ln x) - \int_{x}^{99} e^{t} f(t)\,dt$$

then find $f(x)$ and state its domain.