Name: ____________________
Student ID: ____________________

Test 1

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(10 marks)* Solve the following system by Gauss-Jordan elimination:

$$\begin{array}{ccccccccc} 2x_1 & + & 3x_2 & + & 3x_3 & + & x_4 & = & 1 \\ 3x_1 & + & 2x_2 & - & 2x_3 & + & & = & 2 \\ 5x_1 & + & 5x_2 & + & x_3 & + & x_4 & = & 3 \end{array}$$

In addition, if the solution is not unique give two particular solutions.

Question 2. *(5 marks)* Consider the matrices:

$$A = \begin{bmatrix} 0 & 2 & 1 \\ -1 & 0 & -1 \\ 1 & 1 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 3 & 1 \\ 0 & -4 & 1 \\ 1 & -2 & -1 \end{bmatrix}$$

Then compute $\mathrm{tr}(AA^t - 3B)$, if possible.

Question 3. *(5 marks)* Solve for the matrix X if

$$X \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 2 \\ 5 & 4 & 5 \end{bmatrix}$$

Question 4. *(10 marks)* Solve the following system using the inverse of the coefficient matrix.

$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & + & 2x_3 & = & 0 \\ x_1 & + & 3x_2 & + & 3x_3 & = & -1 \\ 1x_1 & + & 2x_2 & + & 3x_3 & = & 1 \end{array}$$

Question 5. Express

$$A = \begin{bmatrix} -3 & 5 \\ 6 & -2 \end{bmatrix} B = \begin{bmatrix} -3 & 5 \\ 0 & 8 \end{bmatrix}, C = \begin{bmatrix} 0 & 8 \\ -3 & 5 \end{bmatrix}$$

Find the elementary matrices E_1 and E_2 (if possible) such that

a. *(2 marks)* $E_1 A = B$

b. *(2 marks)* $E_2 A = C$

Question 6. *(3 marks)* Show that if A, D are invertible and $ABD = 0$ then $B = 0$.

Question 7. *(3 marks)* Find A^{-1} if $A^4 + 2A - I = 0$.

Question 8. *(5 marks)* If D is an $n \times n$ diagonal matrix, A is an $n \times n$ symmetric matrix and B is an $n \times n$ matrix then show that $B^t B + 2A + D - I$ is symmetric.

Question 9. *(5 marks)* For which value(s) of the constant a and b does the system

$$\begin{array}{rcrcl} x & + & y & = & 1 \\ 9x & + & a^2y & = & b \end{array}$$

have no solutions? Exactly one solution? Infinitely many solutions? Justify.

Bonus Question. *(3 marks)* Prove: If $A^5 = 0$ then

$$(I-A)^{-1} = I + A + A^2 + A^3 + A^4.$$