Dawson College: **Linear Algebra**: 201-NYC-05-S07: Winter 2010

Name: ____________________
Student ID: ____________________

Test 2

This test is graded out of 50 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Use Cramer's rule to solve for x_2 without solving for x_1, x_3.

$$\begin{array}{rcrcrcl} 7x_1 & + & 2x_2 & - & 2x_3 & = & 3 \\ 4x_1 & + & x_2 & & & = & 0 \\ 3x_1 & + & 3x_2 & - & x_3 & = & 1 \end{array}$$

(Use cofactor expansions to find the determinants)

Question 2. *(4 marks)* Show that if A, B are invertible then $\det(A^{-1}BCAB^{-1}) = \det(C)$

Question 3. *(5 marks)*

a. *(5 marks)* Use the combinatorial definition of the determinant to compute:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

b. *(2 marks)* Justify the “visual way” of computing the determinant using part a.

Question 4.

a. *(5 marks)* Find the inverse of the following matrix using the adjoint:

$$A = \begin{bmatrix} 0 & -2 \\ 3 & 1 \end{bmatrix}$$

b. *(2 marks)* Using part a. solve the following equation:

$$Ax = b$$

where

$$b = \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$$

Question 5. *(3 marks)* For which value(s) of α is the following matrix invertible:

$$\begin{bmatrix} \alpha^2-1 & 0 & 0 \\ 1 & \alpha & 0 \\ 2 & -1 & \alpha+1 \end{bmatrix}$$

Question 6. Let

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 9 & 3 & 4 & 1 & 0 & 9 \\ 0 & 1 & 1 & 1 & 4 & 9 & 2 & 7 & 7 & 9 \\ 0 & 0 & 1 & 1 & 1 & 4 & 9 & 2 & 7 & 7 \\ 0 & 0 & 0 & 1 & 1 & 1 & 4 & 9 & 2 & 7 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 4 & 9 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 4 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, C = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, D = \begin{bmatrix} 2b-a & b \\ 2d-c & d \end{bmatrix}$$

a. *(3 marks)* If B is a 10×10 invertible matrix show that AB is invertible.

b. *(4 marks)* If $\det(D) = 2$ then find $\det(C)$.

Question 7. *(5 marks)* Compute the determinant of the following matrix using elementary operations

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 2 & -2 & 1 \\ 6 & 3 & 1 \end{bmatrix}.$$

Question 8. *(5 marks)* If A, B are 3×3 matrices, $\det(2A) = -8$ and $\det(B) = \sqrt{2}$ then find

$$\det((3AB)^t (2AB)^{-1} A^4 B^3 A^{-1}).$$

(show every step)

Question 9.

a. *(1 mark)* Sketch the vector $\mathbf{v} = (3,2,4)$ on a right-handed coordinate system.

b. *(2 marks)* Find a nonzero vector $\mathbf{u}$ with terminal point $Q(3,2,1)$ which is oppositely directed to $\mathbf{v}$.

c. *(2 marks)* Let $\mathbf{a} = (-1,0,2)$, $\mathbf{b} = (0,3,0)$ and $\mathbf{c} = (1,1,0)$. If $\mathbf{w} = 2(\mathbf{a} - \mathbf{b}) + 3\mathbf{c}$ then find a unit vector which has the same direction as $\mathbf{w}$

Bonus Question. *(3 marks)* Consider

$$A = \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix}$$

and use row reduction to show that $\det(A) = (b-a)(c-a)(c-b)$.