Dawson College: **Linear Algebra**: 201-NYC-05-S07: Winter 2010

Name: ____________________
Student ID: ____________________

Test 3

This test is graded out of 43 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Let $\mathbf{u} = (2,-1,0)$, $\mathbf{v} = (-4,0,2)$ and $\mathbf{w} = (2,1,3)$.

a. *(3 marks)* Find the angle between $\mathbf{u}$ and $\mathbf{v}$.

b. *(3 marks)* Find a unit vector orthogonal to both $\mathbf{u}$ and $\mathbf{v}$

c. *(3 marks)* Compute the scalar triple product of $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$.

d. *(1 mark)* Find the volume of the parallelepiped defined by $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$.

Question 2. *(5 marks)* Using projections find the area of the triangle defined by $\mathbf{u} = (1,3,0)$ and $\mathbf{v} = (-2,0,-3)$.

Question 3. *(2 marks)* Find the equation of the line passing through the points: $P_1(-2,1,3)$ and $P_2(1,-2,1)$.

Question 4. *(3 marks)* Find the equation of the plane passing through the points: $P_1(7,-3,3)$, $P_2(2,-1,1)$ and $P_3(0,0,3)$.

Question 5. *(5 marks)* Find the equation of the plane which contains the line $(x,y,z) = (1,1,-1)+t(1,3,4)$ and is parallel to the intersection of $x+3z=1$ and $y+2z=2$.

Question 6.

a. *(2 marks)* Determine if the two planes are parallel: $3x+y-z=10$ and $-9x-3y+3z=-101$.

b. *(2 marks)* Determine if the two planes are perpendicular: $2x-z=101$ and $y+z=-101$.

c. *(2 marks)* Determine if the line and the plane are perpendicular: $(x,y,z)=(1,2,2)+t(2,-1,2)$ and $-2x+3y-6z=1$.

d. *(2 marks)* Does the point $(2,0,-3)$ lie on the line $(x,y,z)=(-1,-2,2)+t(6,4,-10)$.

Question 7. *(5 marks)* Find the distance between the line $(x,y,z) = (-2,2,3) + t(3,0,6)$ and the plane $-2x + y + z = 10$.

Question 8. *(5 marks)* Maximize $P = 4x + 5y$ subject to

$$\begin{aligned} -x + y &\leq 40 \\ 2x - y &\leq 10 \end{aligned}$$

Bonus Question. *(3 marks)* Prove: If $\mathbf{u}$ and $\mathbf{a}$ are non-zero vectors then

$$||\operatorname{proj}_{\mathbf{a}} \mathbf{u}|| = ||\mathbf{u}||\, |\cos\theta|$$

where θ is the angle between $\mathbf{u}$ and $\mathbf{a}$.