Dawson College: Calculus II (SCIENCE): 201-NYB-05-S4: Winter 2011

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Bonus Assignment 1

① Find the Taylor Series of $f(x) = x^3 - 3x^2 + 3x - 1$ about $x = 1$

$f(x) = x^3 - 3x^2 + 3x - 1$ $\qquad$ $f(1) = 1^3 - 3(1)^2 + 3(1) - 1 = 0$

$f'(x) = 3x^2 - 6x + 3$ $\qquad$ $f'(1) = 3(1)^2 - 6(1) + 3 = 0$

$f''(x) = 6x - 6$ $\qquad$ $f''(1) = 6(1) - 6 = 0$

$f'''(x) = 6$ $\qquad$ $f'''(1) = 6$

$f^{(4)}(x) = 0$ $\qquad$ $f^{(4)}(1) = 0$

$\vdots$ $\qquad$ $\vdots$

$f^{(n)}(x) = 0$ $\qquad$ $f^{(n)}(1) = 0 \quad \forall n \geq 4$

$$\therefore f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(1)(x-1)^n}{n!} = \frac{f^{(3)}(1)(x-1)^3}{3!}$$

only non-zero term

$$= \frac{6(x-1)^3}{6}$$

$$= (x-1)^3$$

② Find the Maclaurin Series of $f(x) = \arctan x$.

$f(x) = \arctan x$ $\qquad$ $f(0) = \arctan 0 = 0$

$f'(x) = \dfrac{1}{1+x^2}$ $\qquad$ $f'(0) = 1$

$f''(x) = \dfrac{-2x}{(1+x^2)^2}$ $\qquad$ $f''(0) = 0$

$$f'''(x) = \frac{8x^2}{(1+x^2)^3} - \frac{2}{(1+x^2)^2} \qquad f'''(0) = -2$$

$$f^{(4)}(x) = -\frac{48x^3}{(1+x^2)^4} + \frac{24x}{(1+x^2)^3} \qquad f^{(4)}(0) = 0$$

$$f^{(5)}(x) = \frac{384x^4}{(1+x^2)^5} - \frac{288x^2}{(1+x^2)^4} + \frac{24}{(1+x^2)^3} \qquad f^{(5)}(0) = 24$$

$$f^{(6)}(x) = -\frac{3840x^5}{(1+x^2)^6} + \frac{3840x^3}{(1+x^2)^5} - \frac{720x}{(1+x^2)^4} \qquad f^{(6)}(0) = 0$$

$$f^{(7)}(x) = \frac{46080x^6}{(1+x^2)^7} - \frac{57600x^4}{(1+x^2)^6} + \frac{17280x^2}{(1+x^2)^5} \qquad f^{(7)}(0) = -720$$

$$- \frac{720}{(1+x^2)^7}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n$$

$$= \underbrace{\frac{f^{(0)}(0)}{0!} x^0}_{0} + \frac{f^{(1)}(0)}{1!} x + \underbrace{\frac{f''(0)}{2!} x^2}_{0} + \frac{f'''(0)}{3!} x^3$$

$$+ \underbrace{\frac{f^{(4)}(0)}{4!} x^4}_{0} + \frac{f^{(5)}(0)}{5!} x^5 + \underbrace{\frac{f^{(6)}(0)}{6!} x^6}_{0} + \frac{f^{(7)}(0)}{7!} x^7 + \dots$$

$$= \frac{1}{1!} x + \frac{(-2)}{3!} x^3 + \frac{24}{5!} x^5 + \frac{-720}{7!} x^7 + \dots$$

$$= \frac{1!\, x}{1!} - \frac{2!}{3!} x^3 + \frac{4!}{5!} x^5 - \frac{6!}{7!} x^7 + \dots$$

$$= \underbrace{x}_{a_0} - \underbrace{\frac{x^3}{3}}_{a_1} + \underbrace{\frac{x^5}{5}}_{a_2} - \underbrace{\frac{x^7}{7}}_{a_3} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$$

③ Find the Taylor Series of $f(x) = x^{-2}$ at $x = 1$

$f(x) = x^{-2}$

$f'(x) = -2x^{-3}$

$f''(x) = -2(-3)x^{-4}$

$f'''(x) = -2(-3)(-4)x^{-5}$

$\vdots$

$f^{(n)}(x) = (-1)^n (n+1)!\, x^{-n-2}$

$f(1) = 1^{-2} = 1$

$f'(1) = -2(1)^{-3} = -2$

$f''(1) = 2 \cdot 3$

$f'''(1) = -2 \cdot 3 \cdot 4$

$\vdots$

$f^{(n)}(1) = (-1)^n (n+1)!$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(1)}{n!} (x-1)^n$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (n+1)!}{n!} (x-1)^n$$

$$= \sum_{n=1}^{\infty} (-1)^n (n+1)(x-1)^n$$

④ Find the Maclaurin Series of $f(x) = \sin 2x$

$f(x) = \sin 2x$

$f'(x) = 2\cos 2x$

$f(0) = \sin 0 = 0$

$f'(0) = 2\cos 0 = 2$

$f''(x) = -2^2 \sin 2x \qquad f''(0) = -2^2 \sin 0 = 0$

$f'''(x) = -2^3 \cos 2x \qquad f'''(0) = -2^3 \cos 0 = -2^3$

$f^{(4)}(x) = 2^4 \sin 2x \qquad f^{(4)}(x) = 2^4 \sin 0 = 0$

$f^{(5)}(x) = 2^5 \cos 2x \qquad f^{(5)}(x) = 2^5 \cos 0 = 2^5$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)\, x^n}{n!}$$

$$= \underbrace{\frac{f^{(0)}(0)\, x^0}{0!}}_{0} + \frac{f^{(1)}(0)\, x}{1!} + \underbrace{\frac{f^{(2)}(0)\, x^2}{2!}}_{0} + \frac{f^{(3)}(0)\, x^3}{3!} + \underbrace{\frac{f^{(4)}(0)\, x^4}{4!}}_{0}$$

$$+ \frac{f^{(5)}(0)\, x^5}{5!} + \ldots$$

$$= \underbrace{\frac{2x}{1!}}_{a_0} - \underbrace{\frac{2^3 x^3}{3!}}_{a_1} + \underbrace{\frac{2^5 x^5}{5!}}_{a_2} + \ldots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n\, 2^{2n+1}\, x^{2n+1}}{(2n+1)!}$$