Dawson College: Calculus II (SCIENCE): 201-NYB-05-S4: Winter 2011

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Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(3 marks)* §8.1 #24 Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{\sin 2n}{1+\sqrt{n}}$$

$$b_n = \frac{-1}{1+\sqrt{n}} \leq \frac{\sin 2n}{1+\sqrt{n}} \leq \frac{1}{1+\sqrt{n}} = c_n$$

and $\lim_{n\to\infty} b_n = \lim_{n\to\infty} c_n = 0$

$\therefore$ $a_n \to 0$ as $n \to \infty$ by the Squeeze Theorem.

Question 2. *(2 marks)* §8.2 #15 Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \sqrt[n]{2}$$

Let $a_n = \sqrt[n]{2}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \sqrt[n]{2}$$

$$= \lim_{n\to\infty} 2^{1/n}$$

$$= 2^0 = 1$$

Since $\lim_{n\to\infty} a_n \neq 0$ then the series diverges by n^{th} term divergence test.

Question 3. *(5 marks)* §8.2 #21 Determine whether the series is convergent or divergent by expressing S_n as a telescoping sum. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)} = \sum_{n=1}^{\infty} \left[\frac{1}{n} - \frac{1}{n+3} \right]$$

$$\frac{3}{n(n+3)} = \frac{A}{n} + \frac{B}{n+3}$$

$$\frac{3\cancel{n(n+3)}}{\cancel{n(n+3)}} = \frac{A\,\cancel{n}(n+3)}{\cancel{n}} + \frac{Bn\cancel{(n+3)}}{\cancel{n+3}}$$

$$3 = A(n+3) + Bn$$

Let $n=0$

$$3 = A(0+3) + B(0)$$
$$3 = 3A$$
$$1 = A$$

Let $n=-3$

$$3 = A(-3+3) + B(-3)$$
$$-1 = B$$

$$S_n = a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + \dots + a_{n-6} + a_{n-5} + a_{n-4} + a_{n-3} + a_{n-2} + a_{n-1} + a_n$$

$$= \left[\frac{1}{1} - \cancel{\frac{1}{1+3}}\right] + \left[\frac{1}{2} - \cancel{\frac{1}{2+3}}\right] + \left[\frac{1}{3} - \cancel{\frac{1}{3+3}}\right] + \left[\cancel{\frac{1}{4}} - \cancel{\frac{1}{4+3}}\right] + \left[\cancel{\frac{1}{5}} - \frac{1}{5+3}\right] + \left[\cancel{\frac{1}{6}} - \frac{1}{6+3}\right] + \left[\cancel{\frac{1}{7}} - \frac{1}{7+3}\right] +$$

$$+ \dots + \left[\frac{1}{n-6} - \cancel{\frac{1}{n-6+3}}\right] + \left[\frac{1}{n-5} - \cancel{\frac{1}{n-5+3}}\right] + \left[\frac{1}{n-4} - \cancel{\frac{1}{n-4+3}}\right] + \left[\cancel{\frac{1}{n-3}} - \cancel{\frac{1}{n-3+3}}\right] + \left[\cancel{\frac{1}{n-2}} - \frac{1}{n-2+3}\right] + \left[\cancel{\frac{1}{n-1}} - \frac{1}{n-1+3}\right] + \left[\cancel{\frac{1}{n}} - \frac{1}{n+3}\right]$$

$$= 1 + \frac{1}{2} + \frac{1}{3} - \frac{1}{n+1} - \frac{1}{n+2} - \frac{1}{n+3}$$

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)} = \lim_{n\to\infty} \left[1 + \frac{1}{2} + \frac{1}{3} - \cancel{\frac{1}{n+1}}^{\,0} - \cancel{\frac{1}{n+2}}^{\,0} - \cancel{\frac{1}{n+3}}^{\,0} \right]$$

$$= \frac{11}{6}$$