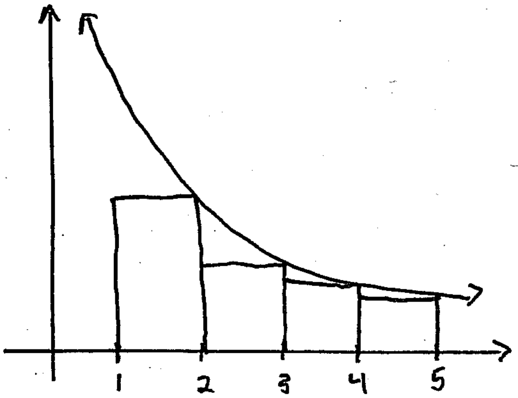
Dawson College: Calculus II (SCIENCE): 201-NYB-05-S4: Winter 2011

Name: Y. Lamontagne
Student ID:

Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §5.1 #3a Estimate the area under the graph of $f(x) = 1/x$ from $x = 1$ to $x = 5$ using four rectangles and right endpoints. Sketch the curve and approximating rectangles. Is your estimate an underestimate or an overestimate?



$n = 4$

$\Delta x = \frac{5-1}{4} = 1$

$x_i = a + i\Delta x = 1 + i$

$x_0 = 1$, $x_1 = 2$, $x_2 = 3$, $x_3 = 4$, $x_4 = 5$

$$\text{Area} \approx f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$$
$$= f(2)\cdot 1 + f(3)\cdot 1 + f(4)\cdot 1 + f(5)\cdot 1$$
$$= \frac{1}{2}\cdot 1 + \frac{1}{3}\cdot 1 + \frac{1}{4}\cdot 1 + \frac{1}{5}\cdot 1$$
$$= \frac{77}{60}$$

$\therefore$ Estimate is an underestimate since function is strictly decreasing.

Question 2. *(5 marks)* §5.2 #20 Use only the definition of the definite integral to evaluate:

$$\int_1^4 (x^2 + 2x - 5)\, dx$$

where $\Delta x = \frac{b-a}{n} = \frac{4-1}{n} = \frac{3}{n}$

$x_i = 1 + \frac{3i}{n}$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(1+\frac{3i}{n}\right)^2 + 2\left(1+\frac{3i}{n}\right) - 5\right]\frac{3}{n}$$
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[1 + \frac{6i}{n} + \frac{9i^2}{n^2} + 2 + \frac{6i}{n} - 5\right]\frac{3}{n}$$
$$= \lim_{n\to\infty} \frac{3}{n}\sum_{i=1}^{n} \left[\frac{9i^2}{n^2} + \frac{12i}{n} - 2\right]$$
$$= \lim_{n\to\infty} \frac{3}{n}\left[\sum_{i=1}^{n}\frac{9i^2}{n^2} + \sum_{i=1}^{n}\frac{12i}{n} - \sum_{i=1}^{n} 2\right]$$
$$= \lim_{n\to\infty} \frac{3}{n}\left[\frac{9}{n^2}\sum_{i=1}^{n} i^2 + \frac{12}{n}\sum_{i=1}^{n} i - 2n\right]$$
$$= \lim_{n\to\infty} \frac{3}{n}\left[\frac{9}{n^{\cancel{2}}}\frac{\cancel{n}(n+1)(2n+1)}{6} + \frac{\cancel{12}^{6}}{\cancel{n}}\frac{\cancel{n}(n+1)}{\cancel{2}} - 2n\right]$$
$$= \lim_{n\to\infty} 3\left[\frac{9}{6}\frac{(n+1)}{n}\frac{(2n+1)}{6n} + \frac{6(n+1)}{n} - \frac{2\cancel{n}}{\cancel{n}}\right]$$
$$= 3\left[\frac{9}{6}\cdot 1\cdot 2 + 6 - 2\right]$$
$$= 3\left[3 + 6 - 2\right] = 21$$