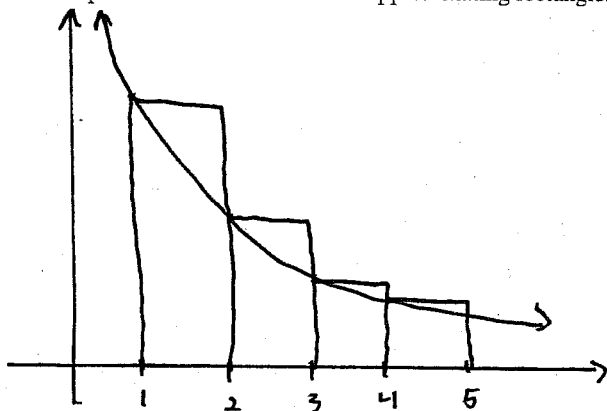


Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §5.1 #3b Estimate the area under the graph of $f(x) = 1/x$ from $x=1$ to $x=5$ using four rectangles and left endpoints. Sketch the curve and approximating rectangles. Is your estimate an underestimate or an overestimate?



$$\begin{aligned} n &= 4 & x_0 &= 1 \\ \Delta x &= \frac{5-1}{4} = 1 & x_1 &= 2 \\ & & x_2 &= 3 \\ x_i &= a + i\Delta x = 1 + i & x_3 &= 4 \\ & & x_4 &= 5 \end{aligned}$$

$$\begin{aligned} \text{Area} &\approx f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x \\ &= f(1) \cdot 1 + f(2) \cdot 1 + f(3) \cdot 1 + f(4) \cdot 1 \\ &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \\ &= \frac{25}{12} \end{aligned}$$

$\therefore$ Estimate is an overestimate since function is strictly decreasing.

Question 2. (5 marks) §5.2 #23 Use only the definition of the definite integral to evaluate:

$$\begin{aligned} \int_1^2 x^3 dx &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x \\ &= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(1 + \frac{i}{n}\right)^3 \frac{1}{n} \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \left(1 + \frac{i}{n}\right)^3 \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n \left[1 + 3\frac{i}{n} + 3\frac{i^2}{n^2} + \frac{i^3}{n^3}\right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[\sum_{i=1}^n 1 + \sum_{i=1}^n \frac{3i}{n} + \sum_{i=1}^n \frac{3i^2}{n^2} + \sum_{i=1}^n \frac{i^3}{n^3} \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[n + \frac{3}{n} \sum_{i=1}^n i + \frac{3}{n^2} \sum_{i=1}^n i^2 + \frac{1}{n^3} \sum_{i=1}^n i^3 \right] \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left[n + \frac{3}{n} \frac{n(n+1)}{2} + \frac{3}{n^2} \frac{n(n+1)(2n+1)}{6} + \frac{1}{n^3} \frac{n^2(n+1)^2}{4} \right] \\ &= \lim_{n \rightarrow \infty} \left[\frac{n}{n} + \frac{3}{2} \frac{(n+1)}{n} + \frac{3(n+1)(2n+1)}{n \cdot 6n} + \frac{(n+1)^2}{4n^2} \right] \\ &= 1 + \frac{3}{2} + 3 \cdot \frac{1}{3} + \frac{1}{4} \\ &= \frac{15}{4} \end{aligned}$$