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# Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** *(5 marks)* §6.2 #7

$$\int_0^{\pi} \sin^4(3t)\,dt = \int_0^{\pi} \sin^2(3t)\sin^2(3t)\,dt$$

$$= \int_0^{\pi} \left(\frac{1-\cos 6t}{2}\right)\left(\frac{1-\cos 6t}{2}\right) dt$$

$$= \frac{1}{4}\int_0^{\pi} 1 - 2\cos 6t + \cos^2 6t\,dt$$

$$= \frac{1}{4}\left[t - \frac{2\sin 6t}{6}\right]_0^{\pi} + \frac{1}{4}\int_0^{\pi} \frac{1+\cos 12t}{2}\,dt$$

$$= \frac{\pi}{4} + \frac{1}{8}\left[t + \frac{\sin 12t}{12}\right]_0^{\pi} = \frac{\pi}{4} + \frac{\pi}{8} = \frac{3\pi}{8}$$

**Question 2.** *(5 marks)* §6.2 #51
Evaluate the integral

$$\int x\sqrt{1-x^4}\,dx = \int \sqrt{1-u^2}\,\frac{du}{2}$$

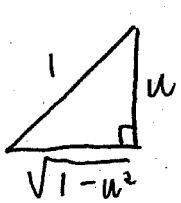
$u = x^2$
$du = 2x\,dx$
$\frac{du}{2} = x\,dx$

$$= \frac{1}{2}\int \sqrt{1-u^2}\,du$$

$u = \sin\theta$
$du = \cos\theta\,d\theta$

$$= \frac{1}{2}\int \sqrt{1-\sin^2\theta}\,\cos\theta\,d\theta$$

$\theta = \arcsin u$




$$= \frac{1}{2}\int \sqrt{\cos^2\theta}\,\cos\theta\,d\theta$$

$$= \frac{1}{2}\int \cos\theta\cos\theta\,d\theta$$

$$= \frac{1}{2}\int \cos^2\theta\,d\theta$$

$$= \frac{1}{2}\int \frac{1+\cos 2\theta}{2}\,d\theta$$

$$= \frac{1}{4}\left[\theta + \frac{\sin 2\theta}{2}\right] + C$$

$$= \frac{1}{4}\left[\arcsin u + \frac{2\sin\theta\cos\theta}{2}\right] + C$$

$$= \frac{1}{4}\left[\arcsin + \sqrt{1-u^2}\,\frac{u}{1}\right] + C$$

$$= \frac{1}{4}\left[\arcsin x^2 + \sqrt{1-x^4}\,x^2\right] + C$$