

Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §6.2 #19

Evaluate the integral.

$$\int \frac{1}{(x+5)^2(x-1)} dx = \int \frac{-1/36}{(x+5)} + \frac{-1/6}{(x+5)^2} + \frac{1/36}{(x-1)} dx = -\frac{1}{36} \ln|x+5| + \frac{1}{6} (x+5)^{-1} + \frac{1}{36} \ln|x-1| + C$$

$$\frac{1}{(x+5)^2(x-1)} = \frac{A}{(x+5)} + \frac{B}{(x+5)^2} + \frac{C}{(x-1)}$$

$$\frac{1 \cdot \cancel{(x+5)^2} (x-1)}{\cancel{(x+5)^2} (x-1)} = \frac{A \cancel{(x+5)^2} (x-1)}{\cancel{(x+5)}} + \frac{B \cancel{(x+5)^2} (x-1)}{\cancel{(x+5)}^2} + \frac{C \cancel{(x+5)^2} (x-1)}{\cancel{(x-1)}}$$

$$1 = A(x+5)(x-1) + B(x-1) + C(x+5)^2$$

Let $x=1$

$$1 = A(1+5)(1-1) + B(1-1) + C(1+5)^2$$

$$1 = 36C \Rightarrow C = 1/36$$

Let $x=-5$

$$1 = A(-5+5)(-5-1) + B(-5-1) + C(-5+5)^2$$

$$1 = -6B \Rightarrow B = -1/6$$

Let $x=0$

$$1 = A(0+5)(0-1) + (-1/6)(0-1) + \frac{1}{36}(0+5)^2$$

$$1 = -5A + 1/6 + \frac{25}{36} \Rightarrow A = -\frac{1}{36}$$

Question 2. (5 marks) §6.6 #32

Determine whether the integral is convergent or divergent. Evaluate if convergent.

$$\int_0^1 \frac{\ln x}{\sqrt{x}} dx = \lim_{a \rightarrow 0^+} \int_a^1 \frac{\ln x}{\sqrt{x}} dx$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$v = 2\sqrt{x}$$

$$dv = \frac{1}{\sqrt{x}} dx$$

$$= \lim_{a \rightarrow 0^+} \left[[uv]_a^1 - \int_a^1 v du \right]$$

$$= \lim_{a \rightarrow 0^+} \left[\left[2 \ln x \sqrt{x} \right]_a^1 - \int_a^1 2\sqrt{x} \frac{1}{x} dx \right]$$

$$= \lim_{a \rightarrow 0^+} \left[\left[\overbrace{2 \ln 1 \sqrt{1}}^0 - 2 \ln a \sqrt{a} \right] - \left[4\sqrt{x} \right]_a^1 \right] \quad \text{I.F. } \infty \cdot 0$$

$$= \lim_{a \rightarrow 0^+} \left[\left[\frac{-2 \ln a}{\frac{1}{\sqrt{a}}} \right] - \left[4\sqrt{1} - 4\sqrt{a} \right] \right] \quad \text{I.F. } \frac{\infty}{\infty}$$

$$= -4 + \lim_{a \rightarrow 0^+} \frac{\frac{-2}{a}}{\frac{-1}{2} \frac{1}{a^{3/2}}} = -4 + \lim_{a \rightarrow 0^+} \frac{4a^{3/2}}{a}$$

$$= -4$$