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Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §6.2 #20
Evaluate the integral.

$$\int \frac{x^2}{(x-3)(x+2)^2}\,dx = \int \frac{9/25}{(x-3)} + \frac{16/25}{(x+2)} + \frac{-4/5}{(x+2)^2}\,dx = \frac{9}{25}\ln|x-3| + \frac{16}{25}\ln|x+2| + \frac{4}{5}(x+2)^{-1} + C$$

$$\frac{x^2}{(x-3)(x+2)^2} = \frac{A}{(x-3)} + \frac{B}{(x+2)} + \frac{C}{(x+2)^2}$$

$$\frac{x^2(x-3)(x+2)^2}{(x-3)(x+2)^2} = \frac{A(x-3)(x+2)^2}{(x-3)} + \frac{B(x-3)(x+2)^2}{(x+2)} + \frac{C(x-3)(x+2)^2}{(x+2)^2}$$

$$x^2 = A(x+2)^2 + B(x-3)(x+2) + C(x-3)$$

Let $x=3$: $3^2 = A(3+2)^2 + B(3-3)(3+2) + C(3-3)$
$9 = 25A \Rightarrow A = 9/25$

Let $x=-2$: $(-2)^2 = A(-2+2)^2 + B(-2-3)(-2+2) + C(-2-3)$
$4 = -5C \Rightarrow C = -4/5$

Let $x=0$: $0^2 = \frac{9}{25}(0+2)^2 + B(0-3)(0+2) + \frac{-4}{5}(0-3)$
$0 = \frac{36}{25} - 6B + \frac{12}{5} \Rightarrow B = \frac{16}{25}$

Question 2. *(5 marks)* §6.6 #31
Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$\int_0^2 z^2 \ln z\,dz = \lim_{a\to 0^+} \int_a^2 z^2 \ln z\,dz$$

$u = \ln z \qquad dw = \frac{1}{z}dz$
$v = \frac{z^3}{3} \qquad dv = z^2 dz$

$$= \lim_{a\to 0^+} \left[[uv]_a^2 - \int_a^2 u\,dv \right]$$

$$= \lim_{a\to 0^+} \left[\left[\frac{z^3}{3}\ln z\right]_a^2 - \int_a^2 \frac{z^3}{3}\frac{1}{z}\,dz \right]$$

I.F. $0\cdot(-\infty)$

$$= \lim_{a\to 0^+} \left[\frac{2^3}{3}\ln 2 - \frac{a^3}{3}\ln a - \frac{1}{3}\left[\frac{z^3}{3}\right]_a^2 \right]$$

$$= \lim_{a\to 0^+} \left[\frac{8}{3}\ln 2 - \frac{\ln a}{3/a^3} - \frac{1}{3}\left[\frac{2^3}{3} - \frac{a^3}{3}\right] \right]$$

I.F. $\frac{\infty}{\infty}$

$$= \frac{8}{3}\ln 2 - \frac{8}{9} - \lim_{a\to 0^+} \frac{1/a}{-9/a^4}$$

$$= \frac{8}{3}\ln 2 - \frac{8}{9} - \lim_{a\to 0^+} \frac{a^{+3}}{-9a} = \frac{8}{3}\ln 2 - \frac{8}{9}$$