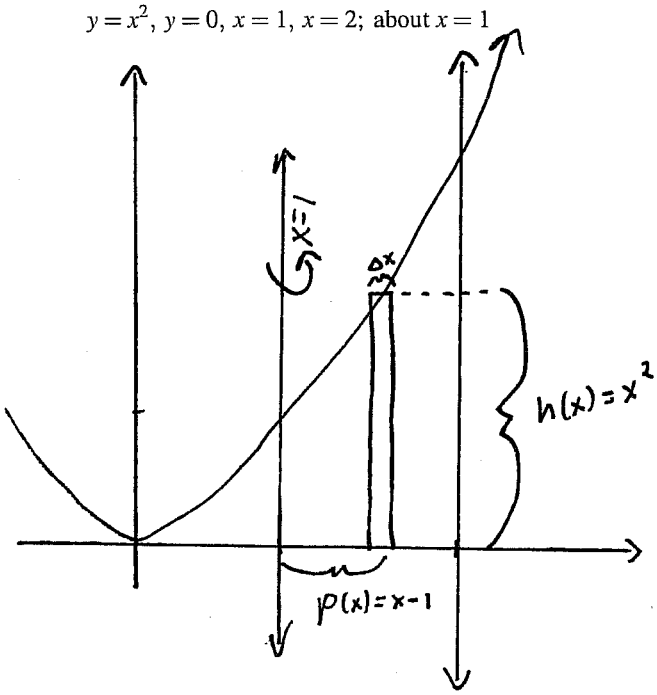
Name: Y. Lamontagne
Student ID: _____

Quiz 9

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §7.3 #15
Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis. Sketch the region and a representative rectangle.

$y = x^2$, $y = 0$, $x = 1$, $x = 2$; about $x = 1$



representative element:

$$\Delta V = 2\pi p(x) h(x) \Delta x$$
$$= 2\pi (x-1) x^2 \Delta x$$

$$V = \int_1^2 2\pi (x-1) x^2 dx$$

$$= 2\pi \int_1^2 x^3 - x^2 \, dx$$

$$= 2\pi \left[\frac{x^4}{4} - \frac{x^3}{3} \right]_1^2$$

$$= 2\pi \left[\left[\frac{2^4}{4} - \frac{2^3}{3} \right] - \left[\frac{1^4}{4} - \frac{1^3}{3} \right] \right]$$

$$= 2\pi \left[\left[4 - \frac{8}{3} \right] - \frac{1}{4} + \frac{1}{3} \right]$$

$$= \frac{17}{6}\pi$$

Question 2. *(2 marks)* §8.1 #5 Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$\left\{1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \cdots\right\}$ $\quad a_n = (-1)^{n+1} \frac{2^{n-1}}{3^{n-1}}$

Question 3. *(3 marks)* §8.1 #26 Determine whether the sequence converges or diverges. If it converges, find the limit.

$a_n = \frac{(\ln n)^2}{n}$ $\quad$ Let $f(x) = \frac{(\ln x)^2}{x}$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{(\ln x)^2}{x} \quad \text{I.F. } \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{2\ln x \cdot \frac{1}{x}}{1} \quad \text{by } H\hat{}$$

$$= \lim_{x\to\infty} \frac{2\ln x}{x} \quad \text{I.F. } \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{2\left(\frac{1}{x}\right)}{1} \quad \text{by } H\hat{}$$

$$= 0$$

$\therefore a_n \to 0$ as $n \to \infty$

Bonus. *(5 marks)* Evaluate the indefinite integral:

$$\int \frac{3x^2+3x+2}{x^3+2x}\,dx$$

See test #2.