

Last Name: SOLUTIONS

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Quiz 3

Question 1. (10 marks) Use the definition of the definite integral to find the following:

$$\int_1^5 (x - 4x^2) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x$$

$$\Delta x = \frac{b-a}{n} = \frac{5-1}{n} = \frac{4}{n} \quad \cdot \quad x_i = a + i \Delta x = 1 + \frac{4i}{n}$$

$$\begin{aligned} \cdot \quad f(x_i) &= x_i - 4(x_i)^2 = \left(1 + \frac{4i}{n}\right) - 4\left(1 + \frac{4i}{n}\right)^2 = 1 + \frac{4i}{n} - 4\left(1 + \frac{8i}{n} + \frac{16i^2}{n^2}\right) \\ &= 1 + \frac{4i}{n} - 4 - \frac{32i}{n} - \frac{64i^2}{n^2} = -3 - \frac{28i}{n} - \frac{64i^2}{n^2} \end{aligned}$$

$$\cdot \quad f(x_i) \Delta x = \left[-3 - \frac{28i}{n} - \frac{64i^2}{n^2}\right] \left[\frac{4}{n}\right] = -\frac{12}{n} - \frac{112i}{n^2} - \frac{256i^2}{n^3}$$

$$\cdot \quad \sum_{i=1}^n f(x_i) \Delta x = \sum_{i=1}^n \left[-\frac{12}{n} - \frac{112i}{n^2} - \frac{256i^2}{n^3}\right] = -\frac{12}{n} \sum_{i=1}^n 1 - \frac{112}{n^2} \sum_{i=1}^n i - \frac{256}{n^3} \sum_{i=1}^n i^2$$

$$= -\frac{12}{n} \cdot n - \frac{112}{n^2} \cdot \frac{n(n+1)}{2} - \frac{256}{n^3} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x = \lim_{n \rightarrow \infty} \left[-12 - 56 \cdot \frac{n+1}{n} - \frac{128}{3} \cdot \frac{(n+1)(2n+1)}{n^2}\right]$$

$$= -12 - 56 \cdot 1 - \frac{128}{3} \cdot 2$$

$$= -68 - \frac{256}{3}$$

$$= -\frac{460}{3}$$