

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 4

Question 1. (4 marks) Find the area of the region under the function $f(x) = 1 - \sqrt[3]{x}$ on the interval $[-8, -1]$. Simplify your final answer.

$$\begin{aligned}
 A &= \int_{-8}^{-1} (1 - x^{1/3}) dx = \left[x - \frac{3}{4} x^{4/3} \right]_{-8}^{-1} = \left[(-1) - \frac{3}{4} (-1)^{4/3} \right] - \left[(-8) - \frac{3}{4} (-8)^{4/3} \right] \\
 &= -1 - \frac{3}{4} + 8 + 12 = \frac{73}{4}
 \end{aligned}$$

Question 1. (3+3 marks) Evaluate the following definite integrals:

$$\begin{aligned}
 \text{(a)} \quad \int_1^4 \left(\sqrt{x} - \frac{1}{\sqrt{x}} \right) dx &= \int_1^4 x^{1/2} - x^{-1/2} dx = \left[\frac{2}{3} x^{3/2} - 2 x^{1/2} \right]_1^4 \\
 &= \left[\frac{2}{3} (4)^{3/2} - 2 (4)^{1/2} \right] - \left[\frac{2}{3} (1)^{3/2} - 2 (1)^{1/2} \right] \\
 &= \frac{16}{3} - 4 - \frac{2}{3} + 2 = \frac{14}{3} - 2 = \frac{8}{3}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad \int_1^4 \frac{3x^3 - 2x^2 + 4}{x^2} dx &= \int_1^4 3x - 2 + 4x^{-2} dx \\
 &= \left[\frac{3x^2}{2} - 2x - 4x^{-1} \right]_1^4 = \left[\frac{3}{2} (4)^2 - 2(4) - \frac{4}{4} \right] - \left[\frac{3}{2} (1)^2 - 2(1) - \frac{4}{1} \right] \\
 &= 24 - 8 - 1 - \frac{3}{2} + 2 + 4 = 21 - \frac{3}{2} = \frac{39}{2}
 \end{aligned}$$