Dawson College: Calculus II - Commerce: 201-NYB-05 S13 March 1, 2012

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 5

Question 1. *(10 marks)* The quantity demanded x (in units of a hundred) of the Mikado miniature cameras/week is related to the unit price p (in dollars) by

$$p = -0.2x^2 + 80$$

and the quantity x (in units of a hundred) that the supplier is willing to make available in the market is related to the unit price p (in dollars) by

$$p = 0.1x^2 + x + 40$$

If the market price is set at the equilibrium price, find the consumers' surplus and the producers' surplus.

$D(x) = -0.2x^2 + 80$, $S(x) = 0.1x^2 + x + 40$

MARKET EQUILIBRIUM:

$D(x) = S(x)$

$-0.2x^2 + 80 = 0.1x^2 + x + 40$

$0 = 0.3x^2 + x - 40$

$0 = 3x^2 + 10x - 400$

$0 = 3x^2 - 30x + 40x - 400$

$0 = 3x(x-10) + 40(x-10)$

$0 = (3x+40)(x-10)$

$\therefore x = -\frac{40}{3}, 10$

$\therefore \bar{x} = 10$

$\bar{p} = D(10) = -0.2(10)^2 + 80 = 60$

$$\therefore CS = \int_0^{10} D(x)\,dx - (10)(60)$$

$$= \int_0^{10} (-0.2x^2 + 80)\,dx - 600$$

$$= \left[-\frac{0.2}{3}x^3 + 80x \right]_0^{10} - 600$$

$$= \left[-\frac{0.2(10)^3}{3} + 80(10) - (0+0) \right] - 600$$

$$= 133.\overline{3}$$

$\therefore$ THE CONSUMER SURPLUS IS $(133.\overline{3})(100) = \$13\,333.33$

$$PS = (60)(10) - \int_0^{10} S(x)\,dx = 600 - \int_0^{10} 0.1x^2 + x + 40\,dx$$

$$= 600 - \left[\frac{0.1x^3}{3} + \frac{x^2}{2} + 40x \right]_0^{10} = 600 - \left[\frac{0.1(10)^3}{3} + \frac{(10)^2}{2} + 40(10) - (0) \right]$$

$$= 116.\overline{6}$$

$\therefore$ THE PRODUCER SURPLUS IS $(116.\overline{6})(100) = \$11\,666.67$