

Last Name: SOLUTIONS

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Quiz 10

Question 1. (4 marks) Find the fourth ($n = 4$) Taylor polynomial for $f(x) = e^{2x}$ at $x = 1$.

$$f(x) = e^{2x}$$

$$f(1) = e^2$$

$$f'(x) = 2e^{2x}$$

$$f'(1) = 2e^2$$

$$f''(x) = 4e^{2x}$$

$$f''(1) = 4e^2$$

$$f'''(x) = 8e^{2x}$$

$$f'''(1) = 8e^2$$

$$f^{(4)}(x) = 16e^{2x}$$

$$f^{(4)}(1) = 16e^2$$

$$P_4(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \frac{f^{(4)}(1)}{4!}(x-1)^4$$

$$= e^2 + 2e^2(x-1) + \frac{4}{2}e^2(x-1)^2 + \frac{8}{6}e^2(x-1)^3 + \frac{16}{24}e^2(x-1)^4$$

$$= e^2 + 2e^2(x-1) + 2e^2(x-1)^2 + \frac{4}{3}e^2(x-1)^3 + \frac{2}{3}e^2(x-1)^4$$

Question 2. (3+3 marks)

a) Find the n th term of the following sequence

$$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \dots = \frac{1}{2^0}, \frac{-1}{2^1}, \frac{1}{2^2}, \frac{-1}{2^3}, \dots$$

$$\therefore a_n = \frac{(-1)^{n+1}}{2^{n-1}}$$

b) Determine if the following sequence converges or diverges. If it converges find its limit:

$$a_n = \frac{2n}{n!}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{2n}{1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot n}$$

$$= \lim_{n \rightarrow \infty} \frac{2}{1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1)}$$

$$= \lim_{n \rightarrow \infty} \frac{2}{(n-1)!} = 0$$