Dawson College: Calculus II - Commerce: 201-NYB-05 S13 March 29, 2012

Last Name: SOLUTIONS

First Name: ____________

Student ID: ____________

Test 2

The length of the test is 1hr and 45min. You may use a nonprogrammable, nongraphing scientific calculator. Remember to clearly show all of your work and to **use correct notation**. Do not use decimals unless otherwise stated.

Question 1. *(6 marks)* James is releasing a new cologne "Jimmy Fresh". He will make x hundred units available in the market when the unit price in the market is $p = \sqrt{144 + 7.2x}$ dollars. Determine the producer's surplus if the market price is \$31/unit. (You may used decimals).

$18 = \sqrt{144 + 7.2x}$

$324 = 144 + 7.2x$

$180 = 7.2x$

$25 = x$

$p = \sqrt{144 + 7.2(25)}$

$= 18$

LET $u = 144 + 7.2x$

$du = 7.2\,dx$

$\frac{du}{7.2} = dx$

IF $x = 0 \Rightarrow u = 144$

$x = 25 \Rightarrow u = 324$

$PS = \bar{p}\bar{x} - \int_0^{\bar{x}} S(x)\,dx = (18)(25) - \int_0^{25} \sqrt{144 + 7.2x}\,dx$

$= 450 - \int_{144}^{324} \sqrt{u}\,\frac{du}{7.2} = 450 - \frac{1}{7.2} \cdot \frac{u^{3/2}}{3/2}\Big|_{144}^{324}$

$= 450 - \frac{2}{21.6}\left[324^{3/2} - 144^{3/2}\right] = 450 - \frac{3}{21.6}\left[5832 - 1728\right]$

$= 450 - 380 = 70$

$\therefore$ THE PRODUCER'S SURPLUS IS $70 \times 100 = \$7000$

Question 2. *(6 marks)* In a study is was found that the Lorentz curve for the distribution of income of paper salespeople is descrived by the function

$$f(x) = \frac{9}{13}x^2 + \frac{4}{13}x$$

and that of beet farmers

$$g(x) = \frac{16}{17}x^4 + \frac{1}{17}x$$

Calculate the coefficient of inequality of each Lorenz curve and determine which professon has a more equitable income distribution. (You may used decimals).

$$L_f = 2\int_0^1 [x - f(x)]\,dx = 2\int_0^1 x - \left(\frac{9}{13}x^2 + \frac{4}{13}x\right)dx = 2\int_0^1 \frac{9}{13}x - \frac{9}{13}x^2\,dx$$

$$= 2 \cdot \frac{9}{13}\int_0^1 (x - x^2)\,dx = \frac{18}{13}\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{18}{13}\left[\frac{1}{2} - \frac{1}{3} - 0\right]$$

$$= \frac{18}{13} \cdot \frac{1}{6} = \frac{3}{13} \approx 0.230769$$

$$L_g = 2\int_0^1 [x - g(x)]\,dx = 2\int_0^1 x - \left(\frac{16}{17}x^4 + \frac{1}{17}x\right)dx$$

$$= 2\int_0^1 \frac{16}{17}x - \frac{16}{17}x^4\,dx = 2 \cdot \frac{16}{17}\int_0^1 x - x^4\,dx$$

$$= \frac{32}{17}\left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1 = \frac{32}{17}\left[\frac{1}{2} - \frac{1}{5} - 0\right] = \frac{32}{17}\left(\frac{3}{10}\right) =$$

$$= \frac{48}{85} \approx 0.564706$$

SINCE $0.230769 < 0.564706$ PAPER SALESPEOPLE HAVE A MORE EQUITABLE INCOME DISTRIBUTION.

Question 3. *(10 marks)* When winning the game show "Know Ya Boo" a couple is presented with two options for prizes. If they choose option 1 they get an immediate payment of \$4000 and then \$100 per week for the next five years. If they choose option 2 they get an income of

$$g(t) = 2600\ t$$

dollars (t is in years) for the next five years, $0 \le t \le 5$. Which option should they choose? (You may used decimals).

PRESENT VALUE OF OPTION 1:

$$PV = \frac{mP}{r}(1 - e^{-rT}) = \frac{(52)(100)}{0.05}(1 - e^{-0.05(5)})$$

$$= 230\ 04.72$$

TOTAL PRESENT VALUE OF OPTION 1 $= 23004.72 + 5000$

$= 27\ 004.72$

PRESENT VALUE OF OPTION 2:

$$PV = \int_0^T R(t)\, e^{-rt}\, dt = \int_0^5 2600\, t e^{-0.05t}\, dt$$

LET

$u = 26000t \qquad dv = e^{-0.05t}\, dt$

$du = 26000\, dt \qquad v = \frac{e^{-0.05t}}{-0.05}$

$$= -20 e^{-0.05t} \cdot 2600\, t \Big|_0^5 - \int_0^5 (2600)(-20 e^{-0.05t})\, dt$$

$$= -52000(5)(e^{-0.25} - 1) + 520\,000 \frac{e^{-0.05t}}{-0.05}\Big|_0^5$$

$$= -520000(5)(e^{-0.25} + 1) - 1040\,000\,(e^{-0.25} - 1)$$

$$= 275\ 58.98$$

Question 4. *(4 marks)* Use the method of partial fractions to write the following as a sum of simpler fractions. You don't have to solve for the variables $A, B, C, \ldots$ etc.

$$\frac{3x^2+2x-1}{x^2(3x-1)^2(x^2+5x+4)(x^2+x+3)^3}$$

NOT IRREDUCIBLE

$$= \frac{3x^2+2x-1}{x^2(3x-1)^2(x+1)(x+4)(x^2+x+3)^3}$$

$$= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{3x-1} + \frac{D}{(3x-1)^2} + \frac{E}{x+1} + \frac{F}{x+4} + \frac{Gx+H}{x^2+x+3} + \frac{Ix+J}{(x^2+x+3)^2}$$

$$+ \frac{Kx+L}{(x^2+x+3)^3}$$

Question 5. Evaluate the following integrals:

(a) *(4 marks)* $\int x\cos x\,dx$

LET

$u = x \qquad dv = \cos x\,dx$

$du = dx \qquad v = \sin x$

$= x\sin x - \int \sin x\,dx$

$= x\sin x + \cos x + C$

(b) *(5 marks)* $\int (e^x - x)^2\,dx \quad = \int e^{2x} - 2xe^x + x^2\,dx$

$= \int e^{2x}\,dx - \int 2xe^x\,dx + \int x^2\,dx$

$= \frac{1}{2}e^{2x} - \left[2xe^x - \int 2e^x\,dx\right] + \frac{x^3}{3}$

$= \frac{1}{2}e^{2x} - 2xe^x + 2e^x + \frac{x^3}{3} + C$

LET

$u = 2x \qquad dv = e^x\,dx$

$du = 2\,dx \qquad v = e^x$

(c) *(8 marks)* $\int \frac{4x^3+24x^2+20x-22}{2x^2+9x-5}\,dx = I$

$$\begin{array}{r} 2x+3 \\ 2x^2+9x-5 \,\big)\, 4x^3+24x^2+20x-22 \\ -(4x^3+18x^2-10x) \\ 6x^2+30x-22 \\ -(6x^2+27x-15) \\ 3x-7 \end{array}$$

FACTOR $2x^2+9x-5 = 2x^2+10x-x-5 = 2x(x+5)-(x+5)$
$= (2x-1)(x+5)$

$$\frac{3x-7}{(2x-1)(x+5)} = \frac{A}{2x-1} + \frac{B}{x+5}$$

$3x-7 = A(x+5) + B(2x-1)$

IF $x = 1/2$

$\frac{3}{2}-7 = A(1/2+5) + B(0)$

$-\frac{11}{2} = \frac{11}{2}A \Rightarrow A = -1$

IF $x = -5$

$-15-7 = A(0) + (-11)B$

$-22 = -11B$

$2 = B$

$$\therefore I = \int 2x+3 - \frac{1}{2x-1} + \frac{2}{x+5}\,dx$$

$$= x^2+3x - \frac{\ln|2x-1|}{2} + 2\ln|x+5| + C$$

(d) *(5 marks)* $\int_1^4 2x \ln\sqrt{x}\,dx$

LET
$u = \ln\sqrt{x} \qquad dv = 2x\,dx$

$du = \frac{1}{\sqrt{x}} \cdot \frac{1}{2}x^{-1/2}dx \qquad v = x^2$

$= \frac{1}{2x}\,dx$

$= (\ln\sqrt{x})\left(\frac{x^2}{2}\right)\Big|_1^4 - \frac{1}{2}\int_1^4 x\,dx$

$= 4^2\ln 2 - \ln 1 - \frac{1}{2}\frac{x^2}{2}\Big|_1^4$

$= 16\ln 2 - \frac{1}{4}16 + \frac{1}{4}$

$= 16\ln 2 - 4 + \frac{1}{4}$

$= 16\ln 2 - \frac{15}{4}$

(e) *(8 marks)* $\displaystyle\int \frac{6x^3-5x^2+2x-3}{x^4+x^2}\,dx$ $= I$ FACTOR: $x^2(x^2+1)$

$$\frac{6x^3-5x^2+2x-3}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1}$$

$$\begin{aligned} 6x^3-5x^2+2x-3 &= Ax(x^2+1) + B(x^2+1) + (Cx+D)x^2 \\ &= Ax^3+Ax + Bx^2+B + Cx^3+Dx^2 \\ &= (A+C)x^3 + (B+D)x^2 + Ax + B \end{aligned}$$

$\therefore$ $6 = A+C$, $-5 = B+D$, $2 = A$, $-3 = B$

$6 = 2 + C$ $\quad -5 = -3 + D$

$4 = C$ $\quad -2 = D$

$$\therefore \quad I = \int \frac{2}{x} - \frac{3}{x^2} + \frac{4x-2}{x^2+1}\,dx$$

$$= 2\ln|x| + \frac{3}{x} + 2\ln|x^2+1| - 2\arctan x + C$$

Question 6. (4 *marks*) Given that $f'(1) = -1$, $f'(2) = 2$, $f(1) = 0$, and $f(2) = 4$, find

$$\int_1^2 x\, f''(x)\,dx$$

LET $u = x$ $\quad dv = f''(x)\,dx$

$du = dx \quad v = f'(x)$

$$= x f'(x)\Big|_1^2 - \int_1^2 f'(x)\,dx$$

$$= x f'(x)\Big|_1^2 - f(x)\Big|_1^2$$

$$= 2 f'(2) - 1 f'(1) - f(2) + f(1)$$

$$= 2(2) - 1(-1) - (4) + 0$$

$$= 4 + 1 - 4 = 1$$