

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 10

Question 1. (3 marks) Find the n th term of the following sequence

$$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \dots = 1, \frac{-1}{2^1}, \frac{1}{2^2}, \frac{-1}{2^3}, \dots$$

$$\therefore a_n = \frac{(-1)^{n+1}}{2^{n-1}}$$

Question 2. (3+4 marks) Determine if the following sequence converges or diverges. If it converges find its limit.

$$\text{a) } a_n = \frac{4n}{n!}$$

$$\begin{aligned} \lim_{n \rightarrow \infty} a_n &= \lim_{n \rightarrow \infty} \frac{4n}{n!} = \lim_{n \rightarrow \infty} \frac{4\cancel{n}}{1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) \cdot \cancel{n}} \\ &= \lim_{n \rightarrow \infty} \frac{4}{1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1)} = \lim_{n \rightarrow \infty} \frac{4}{(n-1)!} \\ &= 0 \quad (\text{converges}) \end{aligned}$$

$$\text{b) } a_n = \frac{e^{5n} + n}{e^{2n}}$$

$$\begin{aligned} \lim_{n \rightarrow \infty} a_n &= \lim_{n \rightarrow \infty} \frac{e^{5n} + n}{e^{2n}} = \text{l.f. } \frac{\infty}{\infty} \stackrel{\textcircled{H}}{=} \lim_{n \rightarrow \infty} \frac{5e^{5n} + 1}{2e^{2n}} = \text{l.f. } \frac{\infty}{\infty} \\ &\stackrel{\textcircled{H}}{=} \lim_{n \rightarrow \infty} \frac{25e^{5n}}{4e^{2n}} = \lim_{n \rightarrow \infty} \frac{25}{4} e^{3n} = \infty \quad (\text{DIVERGES}) \end{aligned}$$