Last Name: SOLUTIONS

First Name:

Student ID:

Quiz 3 (A)

Question 1. *(10 marks)* Use the definition of the definite integral to find the following:

$$\int_{-5}^{-1} (x^2+3x+5)\,dx$$

• $\Delta x = \frac{b-a}{n} = \frac{-1-(-5)}{n} = \frac{4}{n}$ • $x_i = a + i\Delta x = -5 + \frac{4i}{n}$

• $f(x_i) = x_i^2 + 3x_i + 5 = \left(-5+\frac{4i}{n}\right)^2 + 3\left(-5+\frac{4i}{n}\right)+5$

$= 25 - \frac{40i}{n} + \frac{16i^2}{n^2} - 15 + \frac{12i}{n} + 5 = 15 - \frac{28i}{n} + \frac{16i^2}{n^2}$

• $f(x_i)\Delta x = \left(15 - \frac{28i}{n} + \frac{16i^2}{n^2}\right)\left(\frac{4}{n}\right) = \frac{60}{n} - \frac{112i}{n^2} + \frac{64i^2}{n^3}$

• $\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}\left(\frac{60}{n} - \frac{112i}{n^2} + \frac{64i^2}{n^3}\right) = \frac{60}{n}\sum_{i=1}^{n} 1 - \frac{112}{n^2}\sum_{i=1}^{n} i + \frac{64}{n^3}\sum_{i=1}^{n} i^2$

$= \frac{60}{n}\cdot n - \frac{112}{n^2}\cdot\frac{n(n+1)}{2} + \frac{64}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$

• $\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\left[60 - \frac{112}{2}\cdot\frac{n+1}{n} + \frac{64}{6}\cdot\frac{(n+1)(2n+1)}{n^2}\right]$

$= 60 - 56\cdot\frac{1+0}{1} + \frac{32}{3}\cdot\frac{2+0+0}{1}$

$= 60 - 56 + \frac{64}{3} = \frac{76}{3}$