Last Name: SOLUTIONS
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Quiz 3 (B)

Question 1. *(10 marks)* Use the definition of the definite integral to find the following:

$$\int_{-2}^{3}(1-5x^2)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

$$\bullet\ \Delta x = \frac{b-a}{n} = \frac{3-(-2)}{n} = \frac{5}{n} \qquad \bullet\ x_i = a + i\Delta x = -2 + \frac{5i}{n}$$

$$\bullet\ f(x_i) = 1 - 5x_i^2 = 1 - 5\left(-2+\frac{5i}{n}\right)^2 = 1 - 5\left[4 - \frac{20i}{n} + \frac{25i^2}{n^2}\right]$$

$$= 1 - 20 + \frac{100i}{n} - \frac{125i^2}{n^2} = -19 + \frac{100i}{n} - \frac{125i^2}{n^2}$$

$$f(x_i)\Delta x = \left[-19 + \frac{100i}{n} - \frac{125i^2}{n^2}\right]\left(\frac{5}{n}\right) = -\frac{95}{n} + \frac{500i}{n^2} - \frac{625i^2}{n^3}$$

$$\sum_{1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}\left[-\frac{95}{n} + \frac{500i}{n^2} - \frac{625i^2}{n^3}\right] =$$

$$= -\frac{95}{n}\sum_{i=1}^{n}1 + \frac{500}{n^2}\sum_{i=1}^{n}i - \frac{625}{n^3}\sum_{i=1}^{n}i^2 = -\frac{95}{n}\cdot n + \frac{500}{n^2}\cdot\frac{n(n+1)}{2} - \frac{625}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$$

$$\bullet\ \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\left[-95 + 250\cdot\frac{n+1}{n} - \frac{625}{6}\cdot\frac{2n^2+3n+1}{n^2}\right]$$

$$= -95 + 250\cdot\frac{1+0}{1} - \frac{625}{3}\cdot\frac{2+0+0}{0}$$

$$= -95 + 250 - \frac{625}{3}$$

$$= -\frac{160}{3}$$

$$\int_{-2}^{3}(1-5x^2)\,dx = -\frac{160}{3}$$