

Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Quiz 4 (B)

Question 1. Find the following. Simplify your final answer.

(a) (5 marks) $\int_1^5 x\sqrt{x-1} dx$

$$= \int_0^4 (u+1)\sqrt{u} du = \int_0^4 (u^{3/2} + u^{1/2}) du$$

$$= \left[\frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} \right]_0^4 = \left[\frac{2}{5} (4)^{5/2} + \frac{2}{3} (4)^{3/2} \right] - [0 + 0]$$

$$= \frac{2}{5} (32) + \frac{2}{3} (8)$$

$$\begin{aligned} \text{Let } u &= x-1 \Rightarrow x = u+1 \\ du &= dx \\ \text{If } x=1 &\Rightarrow u=0 \\ x=5 &\Rightarrow u=4 \end{aligned}$$

$$= \frac{192}{5} + \frac{16}{3} = \frac{272}{15}$$

(b) (5 marks) $\int_0^{\pi/2} \frac{\cos x}{1 + \sin x} dx$

$$= \int_1^2 \frac{\cos x}{u} \frac{du}{\cos x} = \int_1^2 \frac{1}{u} du$$

$$= \ln|u| \Big|_1^2 = \ln 2 - \ln 1 = \ln 2$$

$$\begin{aligned} \text{Let } u &= 1 + \sin x \\ du &= \cos x dx \\ \frac{du}{\cos x} &= dx \end{aligned}$$

$$\text{If } x=0 \Rightarrow u = 1 + \sin(0) = 1$$

$$x = \pi/2 \Rightarrow u = 1 + \sin \frac{\pi}{2} = 2$$