

Last Name: SOLUTIONS

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Quiz 5

Question 1. (10 marks) The quantity demanded x (in units of a hundred) of the Mikado miniature cameras/week is related to the unit price p (in dollars) by

$$p = -0.2x^2 + 80$$

and the quantity x (in units of a hundred) that the supplier is willing to make available in the market is related to the unit price p (in dollars) by

$$p = 0.1x^2 + x + 40$$

If the market price is set at the equilibrium price, find the consumers' surplus and the producers' surplus.

$$D(x) = -0.2x^2 + 80, \quad S(x) = 0.1x^2 + x + 40$$

MARKET EQUILIBRIUM:

$$D(x) = S(x)$$

$$-0.2x^2 + 80 = 0.1x^2 + x + 40$$

$$0 = 0.3x^2 + x - 40$$

$$0 = 3x^2 + 10x - 400$$

$$0 = 3x^2 - 30x + 40x - 400$$

$$0 = 3x(x - 10) + 40(x - 10)$$

$$0 = (3x + 40)(x - 10)$$

$$\therefore x = \frac{-40}{3}, 10$$

$$\therefore \bar{x} = 10$$

$$p = D(10) = -0.2(10)^2 + 80 = 60$$

$$\therefore CS = \int_0^{10} D(x) dx - (10)(60)$$

$$= \int_0^{10} (-0.2x^2 + 80) dx - 600$$

$$= \left[-\frac{0.2}{3}x^3 + 80x \right]_0^{10} - 600$$

$$= \left[-\frac{0.2(10)^3}{3} + 80(10) - (0 + 0) \right] - 600$$

$$= 133.\bar{3}$$

$\therefore$ THE CONSUMER SURPLUS IS
 $(133.\bar{3})(100) = \$13\,333.33$

$$PS = (60)(10) - \int_0^{10} S(x) dx = 600 - \int_0^{10} 0.1x^2 + x + 40 dx$$

$$= 600 - \left[\frac{0.1x^3}{3} + \frac{x^2}{2} + 40x \right]_0^{10} = 600 - \left[\frac{0.1(10)^3}{3} + \frac{(10)^2}{2} + 40(10) - (0) \right]$$

$$= 116.\bar{6} \quad \therefore \text{THE PRODUCER SURPLUS IS}$$

$$(116.\bar{6})(100) = \$11\,666.67$$