

Last Name: SOLUTIONS

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Quiz 9

Question 1. (5 marks) Find the fourth ($n = 4$) Taylor polynomial for $f(x) = \ln x$ at $x = 1$.

$$f(x) = \ln x$$

$$f(1) = \ln 1 = 0$$

$$f'(x) = \frac{1}{x}$$

$$f'(1) = 1$$

$$f''(x) = -\frac{1}{x^2}$$

$$f''(1) = -1$$

$$f'''(x) = \frac{2}{x^3}$$

$$f'''(1) = 2$$

$$f^{(4)}(x) = -\frac{6}{x^4}$$

$$f^{(4)}(1) = -6$$

$$\therefore T_4(x) = f(1) + f'(1)(x-1) + \frac{f''(1)}{2}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \frac{f^{(4)}(1)}{4!}(x-1)^4$$

$$= 0 + (x-1) - \frac{1}{2}(x-1)^2 + \frac{2}{6}(x-1)^3 - \frac{6}{24}(x-1)^4$$

$$= (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$$

Question 2. (5 marks) Solve the first-order differential equation:

$$y' = \frac{y \ln x}{x}$$

$$\frac{dy}{dx} = \frac{y \ln x}{x} \Rightarrow \frac{1}{y} dy = \frac{\ln x}{x} dx$$

$$\int \frac{1}{y} dy = \int \frac{\ln x}{x} dx = \int u du$$

$$\begin{array}{l} \text{Let } u = \ln x \\ du = \frac{1}{x} dx \end{array}$$

$$\ln|y| = \frac{u^2}{2} + C$$

$$\ln|y| = \frac{(\ln x)^2}{2} + C$$

$$e^{\ln|y|} = e^{\frac{(\ln x)^2}{2} + C}$$

$$|y| = e^{\frac{(\ln x)^2}{2}} \cdot e^C$$

$$|y| = |C_1| e^{\frac{(\ln x)^2}{2}}$$

$$y = \pm |C_1| e^{\frac{(\ln x)^2}{2}}$$

$$y = C e^{\frac{(\ln x)^2}{2}}$$