**Dawson College**: **Calculus II - Commerce**: 201-NYB-05 S14 February 27, 2012

**Last Name:** SOLUTIONS

**First Name:** ____________

**Student ID:** ____________

# Test 1 (A)

The length of the test is 1hr and 45min. You may use a nonprogrammable, nongraphing scientific calculator. Remember to clearly show all of your work and to **use correct notation**. Do not use decimals unless otherwise stated.

**Question 1.** Find/evaluate the following:

(a) *(3 marks)* $\int \frac{x^3-2x+5}{x^2}\,dx$ $= \int \left(x - \frac{2}{x} + 5x^{-2}\right)dx$

$= \frac{x^2}{2} - 2\ln|x| - \frac{5}{x} + C$

(b) *(4 marks)* $\int \frac{e^{-1/x}}{x^2}\,dx$

LET $u = -\frac{1}{x}$

$du = \frac{1}{x^2}dx$

$x^2 du = dx$

$= \int \frac{e^u}{x^2} x^2 du = \int e^u du$

$= e^u + C$

$= e^{-1/x} + C$

(c) *(4 marks)* $\int \frac{3x^2-3x+5}{2x^3-3x^2+10x}\,dx$

LET $u = 2x^3 - 3x^2 + 10x$

$du = (6x^2 - 6x + 10)dx$

$\frac{du}{2(3x^2-3x+5)} = dx$

$$= \int \frac{3x^2-3x+5}{u} \cdot \frac{du}{2(3x^2-3x+5)}$$

$$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|2x^3-3x^2+10x| + C$$

(d) *(4 marks)* $\int \frac{(\sqrt{x}-x)^2}{x}\,dx$ $= \int \frac{x - 2x^{3/2} + x^2}{x}\,dx$

$$= \int (1 - 2x^{1/2} + x)\,dx = x - 2\left(\frac{2}{3}x^{3/2}\right) + \frac{x^2}{2} + C$$

$$= x - \frac{4}{3}x^{3/2} + \frac{1}{2}x^2 + C$$

**Question 2.** *(5 marks)* Let $f'(x) = \sqrt{x} + \frac{1}{x}$. Find $f(x)$ given $f(9) = 3$.

$$f(x) = \int f'(x)\,dx = \int \left(\sqrt{x} + \frac{1}{x}\right)dx = \frac{2}{3}x^{3/2} + \ln|x| + c$$

$$3 = f(9) = \frac{2}{3}(9)^{3/2} + \ln(9) + c$$

$$= \frac{2}{3}(27) + \ln 9 + c$$

$$3 - 18 - \ln 9 = c$$

$$-15 - \ln 9 = c$$

$$\therefore \quad f(x) = \frac{2}{3}x^{3/2} + \ln|x| - 15 - \ln 9$$

**Question 3.**
**(a)** *(1 marks)* Write the definition of the definite integral

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

**(b)** *(3 marks)* Using **words** describe
**i)** What $\Delta x$ is.

$\Delta x$ IS THE WIDTH OF AN APPROXIMATING RECTANGLE IN THE RIEMANN SUM.

**ii)** What $f(x_i)\Delta x$ is.

$f(x_i)\Delta x$ IS THE AREA OF AN APPROXIMATING RECTANGLE ON THE SUBINTERVAL $[x_{i-1}, x_i]$ IN THE RIEMANN SUM.

**iii)** What the purpose of the limit in the definition is.

THE SUM OF APPROXIMATING RECTANGLES $\sum_{i=1}^{n} f(x_i)\Delta x$ IS AN APPROXIMATION OF THE "NET AREA" UNDER THE FUNCTION $f(x)$ ON $[a,b]$ THIS APPROXIMATION GETS BETTER AS $n$ GETS LARGER (MORE RECTANGLES). THE LIMIT IN THE DEFINITION GIVES US THE EXACT AREA.

**(c)** *(6 marks)* Use the definition of the definite integral to find:

$$\int_1^4 (2-2x+x^2)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

- $\Delta x = \frac{b-a}{n} = \frac{4-1}{n} = \frac{3}{n}$ • $x_i = a + i\Delta x = 1 + \frac{3i}{n}$

- $f(x_i) = 2 - 2x_i + x_i^2 = 2 - 2\left(1+\frac{3i}{n}\right) + \left(1+\frac{3i}{n}\right)^2$

$$= 2 - 2 - \frac{6i}{n} + 1 + \frac{6i}{n} + \frac{9i^2}{n^2} = 1 + \frac{9i^2}{n^2}$$

- $f(x_i)\Delta x = \left(1+\frac{9i^2}{n^2}\right)\left(\frac{3}{n}\right) = \frac{3}{n} + \frac{27i^2}{n^3}$

- $\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}\left(\frac{3}{n} + \frac{27i^2}{n^3}\right) = \frac{3}{n}\sum_{i=1}^{n} 1 + \frac{27}{n^3}\sum_{i=1}^{n} i^2$

$$= \frac{3}{n}\cdot n + \frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} = 3 + \frac{9}{2}\cdot\frac{2n^2+3n+1}{n^2}$$

- $\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x = \lim_{n\to\infty}\left[3 + \frac{9}{2}\cdot\frac{2+3/n+1/n^2}{1}\right]$

$$= 3 + \frac{9}{2}\cdot\frac{2+0+0}{1} = 3+9 = 12$$

$$\therefore \int_1^4 (2-2x+x^2)\,dx = 12$$

**Question 4.** Evaluate the following definite integrals:

(a) *(5 marks)* $\int_{\pi/8}^{\pi/4} \csc^2 2x\,dx$

LET $u = 2x$

$du = 2dx$

$x = \frac{\pi}{4} \Rightarrow u = \pi/2$

$x = \pi/8 \Rightarrow u = \pi/4$

$$= \int_{\pi/4}^{\pi/8} \csc^2 u \frac{du}{2} = \frac{1}{2}\int_{\pi/4}^{\pi/8} \csc^2 u\,du$$

$$= -\frac{1}{2}\cot u \Big|_{\pi/4}^{\pi/2} = \left(-\frac{1}{2}\cot \pi/2\right) - \left(-\frac{1}{2}\cot \pi/4\right)$$

$$= -\frac{0}{2} + \frac{1}{2}$$

$$= 1/2$$

(b) *(5 marks)* $\int_{2}^{10} \frac{x}{\sqrt{x-1}}\,dx$

LET $u = x - 1 \Rightarrow u + 1 = x$

$du = dx$

IF $x = 2 \Rightarrow u = 1$

$x = 10 \Rightarrow x = 9$

$$= \int_1^9 \frac{u+1}{\sqrt{u}}\,du = \int_1^9 u^{1/2} + u^{-1/2}\,du$$

$$= \left[\frac{2}{3}u^{3/2} + 2u^{1/2}\right]_1^9$$

$$= \left[\frac{2}{3}(9)^{3/2} + 2(9)^{1/2}\right] - \left[\frac{2}{3}(1)^{3/2} + 2(1)^{1/2}\right]$$

$$= 18 + 6 - \frac{2}{3} - 2 = 22 - \frac{2}{3} = \frac{64}{3}$$

**Question 5.** *(4 marks)* Find the area of the region bounded between $f(x) = x^3 + x^2 - 5x + 2$ and $g(x) = x + 2$. Sketch a graph to help solve the problem.

INTERSECTION POINTS:

$f(x) = g(x)$

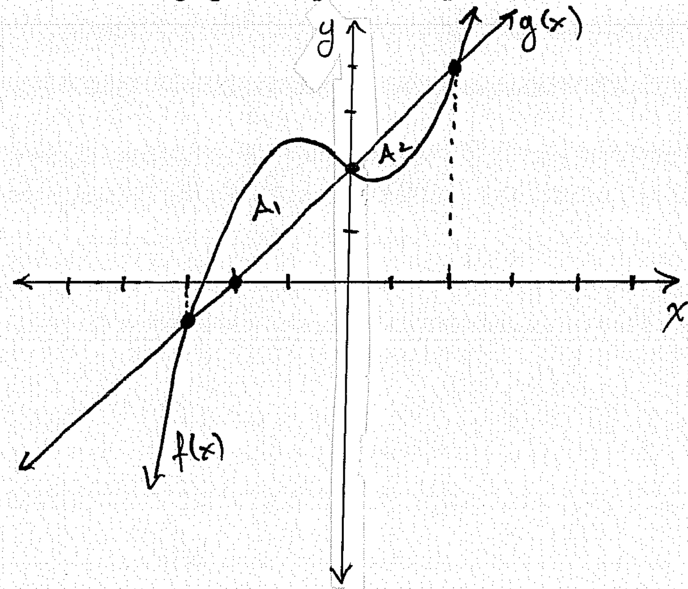
$x^3 + x^2 - 5x + 2 = x + 2$

$x^3 + x^2 - 6x = 0$

$x(x^2 + x - 6) = 0$

$x(x+3)(x-2) = 0$

$\therefore x = -3, 0, 2$




$$\therefore A = A_1 + A_2 = \int_{-3}^{0} [f(x) - g(x)]\,dx + \int_{0}^{2} [g(x) - f(x)]\,dx$$

$$= \int_{-3}^{0} (x^3 + x^2 - 5x + 2) - (x+2)\,dx + \int_{0}^{2} (x+2) - (x^3 + x^2 - 5x + 2)\,dx$$

$$= \int_{-3}^{0} (x^3 + x^2 - 6x)\,dx + \int_{0}^{2} (6x - x^2 - x^3)\,dx = \left[\frac{x^4}{4} + \frac{x^3}{3} - 3x^2\right]_{-3}^{0} + \left[3x^2 - \frac{x^3}{3} - \frac{x^4}{4}\right]_{0}^{2}$$

$$0 - \left(\frac{(-3)^4}{4} + \frac{(-3)^3}{3} - 3(-3)^2\right)] + \left[\left(3(2)^2 - \frac{(2)^3}{3} - \frac{(2)^4}{4}\right) - (0)\right]$$

$$= -\frac{81}{4} + 9 + 27 + 12 - \frac{8}{3} - 4 = -\frac{81}{4} - \frac{8}{3} + 44 = \frac{253}{12}$$

**Question 6.** *(5 marks)* Given

$$\int_1^5 f(x)\,dx = 10, \quad \int_5^7 f(x)\,dx = -2 \text{ and } \int_5^9 f(x)\,dx = 7$$

find the following (show your work for full marks):

(a) $\int_1^9 f(x)\,dx$

$$= \int_1^5 f(x)dx + \int_5^9 f(x)dx$$

$$= 10 + 7 = 17$$

(b) $\int_7^9 f(x)\,dx$

$$= \int_5^9 f(x)dx - \int_5^7 f(x)\,dx$$

$$= 7 - (-2) = 9$$

(c) $\int_5^1 f(x)\,dx$

$$= -\int_1^5 f(x)\,dx = -10$$

(d) $\int_1^6 f(x)\,dx + \int_6^7 f(x)\,dx$

$$= \int_1^7 f(x)dx$$

$$= \int_1^5 f(x)dx + \int_5^7 f(x)dx$$

$$= 10 + (-2)$$

$$= 8$$