Dawson College: **Calculus II - Commerce**: 201-NYB-05 S14

April 2, 2012

Last Name: SOLUTIONS

First Name: ____________

Student ID: ____________

Test 2

The length of the test is 1hr and 45min. You may use a nonprogrammable, nongraphing scientific calculator. Remember to clearly show all of your work and to **use correct notation**. Do not use decimals unless otherwise stated.

Question 1. *(7 marks)* The quantity demanded x (in units of a hundred) of a product/week is related to the unit price p (in dollars) by

$$p = -0.1x^2 + 70$$

and the quantity x (in units of a hundred) that the supplier is willing to make available in the market is related to the unit price p (in dollars) by

$$p = 0.1x^2 + x + 40$$

If the market price is set at the equilibrium price , find the producers' surplus.

$S(x) = 0.1x^2 + x + 40 = -0.1x^2 + 70 = D(x)$

$0.2x^2 + x - 30 = 0$ $\qquad\qquad \bar{p} = -0.1(10)^2 + 70 = 60$

$2x^2 + 10x - 300 = 0$

$x^2 + 5x - 150 = 0$

$(x-10)(x+15) = 0$

$\therefore \bar{x} = 10 \qquad \cancel{x = -15}$

$\therefore \quad PS = \bar{p}\bar{x} - \int_0^{\bar{x}} S(x)\,dx = (60)(10) = \int_0^{10} (0.1x^2 + x + 40)\,dx$

$= 600 - \left[\frac{0.1x^3}{3} + \frac{x^2}{2} + 40x\right]_0^{10} = 600 - \left[\frac{0.1(10)^3}{3} + \frac{(10)^2}{2} + 40(10) - 0\right]$

$= 600 - 483.\overline{3} = \quad 483.\overline{3}$

$\therefore$ THE PRODUCERS' SURPLUS IS $(116.\overline{6})(100) = \$11\ 666.67$

Question 2. *(8 marks)* Emilie wants to get a part time job for two years in order to buy a car. She budgets that, two years from now, the price of the car will be \$21 600. She has two options for part time jobs.

Option 1 pays an income of \$$10000t$ dollars per year where t is in years, $0 \le t \le 2$. If she stays at this job for two years they will give her a bonus of \$1000 in two years.

Option 2 pays \$200 per week for the next two years.

Assuming she saves the money she earns at an interest rate of 4% per year compounded continuously which option should she choose? Will she be able to buy the car?

FUTURE VALUE OF OPTION 1

$$A = e^{rT}\int_0^T R(t)e^{-rt}\,dt = e^{0.04(2)}\int_0^2 10000te^{-0.04t}\,dt$$

LET $u = t \qquad dv = e^{-0.04t}dt$

$du = dt \qquad v = \frac{e^{-0.04t}}{-0.04}$

$$= 10000e^{0.08}\int_0^2 te^{-0.04t}\,dt = 10000e^{0.08}\left[uv\Big|_0^2 - \int_0^2 v\,du\right]$$

$$= 10000e^{0.08}\left[-25te^{-0.04t}\Big|_0^2 + \int_0^2 25e^{-0.04t}\,dt\right]$$

$$= 10000e^{0.08}\left[-50e^{-0.08} + 0 + \left[-25^2e^{-0.04t}\right]_0^2\right]$$

$$= 10000e^{0.08}\left[-50e^{-0.08} - 625e^{-0.08} + 625\right] = \$20\,544.17$$

FUTURE VALUE: \$20 544.17 + \$1000 = \$21 544.17

FUTURE VALUE OF OPTION 2

$$A = \frac{mP}{r}(e^{rT} - 1) = \frac{52(200)}{0.04}\left(e^{0.04(2)} - 1\right) = \$21\,654.64$$

SHE SHOULD CHOOSE OPTION 2. SHE WILL BE ABLE TO BUY THE CAR.

Question 3. *(5 marks)* In a study is was found that the Lorentz curve for the distribution of income of circus clowns is descrived by the function

$$f(x) = \frac{9}{11}x^2 + \frac{2}{11}x$$

Calculate the coefficient of inequality of this Lorenz curve. If it is known that the Lorenz curve for acrobats has a coefficient of inequality of 0.256 which profession has a more equitable income distribution. (You may used decimals).

$$L = 2\int_0^1 [x - f(x)]\,dx = 2\int_0^1 \left[x - \left(\frac{9}{11}x^2 + \frac{2}{11}x\right)\right] dx$$

$$= 2\int_0^1 \left(\frac{9}{11}x - \frac{9}{11}x^2\right) dx = \frac{18}{11}\int_0^1 (x - x^2)\,dx$$

$$= \frac{18}{11}\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{18}{11}\left[\frac{1}{2} - \frac{1}{3} + 0\right]$$

$$= \frac{18}{11}\left[\frac{1}{6}\right] = \frac{3}{11} \approx 0.\overline{27}$$

SINCE $0.\overline{27} > 0.256$, ACROBATS HAVE A MORE EQUITABLE INCOME DISTRIBUTION.

Question 4. *(4 marks)* Use the method of partial fractions to write the following as a sum of simpler fractions. **You don't have to solve for the variables** $A, B, C, \ldots$ etc.

$$\frac{3x^2+2x-1}{x^2(3x-1)^2(x^2+5x+4)(x^2+x+3)^3}$$

IRREDUCIBLE: $1^2-4(1)(3)<0$

NOT IRREDUCIBLE

$x^2+5x+4 = (x+4)(x+1)$

$$\therefore \frac{3x^2+2x-1}{x^2(3x-1)^2(x+4)(x+1)(x^2+x+3)^3}$$

$$= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+4} + \frac{D}{x+1} + \frac{E}{(3x-1)} + \frac{F}{(3x-1)^2} + \frac{Gx+H}{x^2+x+3} + \frac{Ix+J}{(x^2+x+3)^2} + \frac{Kx+L}{(x^2+x+3)^3}$$

Question 5. Evaluate the following integrals:

(a) *(4 marks)* $\int x\sin x\,dx = uv - \int v\,du$

LET $u = x$ $\quad dv = \sin x\,dx$

$du = dx$ $\quad v = -\cos x$

$$= -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x + C$$

(b) *(5 marks)* $\int_0^2 (x-4)e^{2x+1}\,dx$

LET

$u = x - 4$ $\quad dv = e^{2x+1}\,dx$

$du = dx$ $\quad v = \dfrac{e^{2x+1}}{2}$

$$= uv\Big|_0^2 - \int_0^2 v\,du$$

$$= (x-4)\frac{e^{2x+1}}{2}\Big|_0^2 - \int_0^2 \frac{e^{2x+1}}{2}\,dx$$

$$= \frac{-2e^5}{2} + \frac{4e^1}{2} - \frac{e^{2x+1}}{4}\Big|_0^2 = -e^5 + 2e - \frac{e^5}{4} + \frac{e}{4}$$

$$= \frac{-5e^4}{4} + \frac{9e}{4}$$

(c) (8 marks) $\displaystyle I = \int \frac{6x^3+31x^2+2x+17}{3x^2+14x-5}\,dx = \int 2x+1 + \frac{-7x+22}{3x^2+14x-5}\,dx$

$$\begin{array}{r}
2x+1 \\
3x^2+14x-5 \;\big)\; 6x^3+31x^2+2x+17 \\
-(6x^3+28x^2-10x) \\
\hline
3x^2+12x+17 \\
-(3x^2+14x-5) \\
\hline
-2x+22
\end{array}$$

FACTOR: $3x^2+14x-5$

$= 3x^2+15x-x-5$

$= 3x(x+5)-(x+5)$

$= (3x-1)(x+5)$

$$\frac{-2x+22}{(3x-1)(x+5)} = \frac{A}{3x-1} + \frac{B}{x+5}$$

$-2x+22 = A(x+5) + B(3x-1)$

IF $x=-5$

$-2(-5)+22 = A(0) + B(-16)$

$32 = -16B$

$-2 = B$

IF $x = \frac{1}{3}$

$-2(\frac{1}{3})+22 = A(\frac{1}{3}+5) + B(0)$

$\frac{64}{3} = \frac{16A}{3}$

$4 = A$

$$\therefore I = \int 2x+1 + \frac{4}{3x-1} - \frac{2}{x+5}\,dx$$

$$= x^2 + x + \frac{4}{3}\ln|3x-1| - 2\ln|x+5| + C$$

(d) *(5 marks)* $\int (\ln x)^2\, dx$

LET

$u = (\ln x)^2 \qquad dv = dx$

$du = 2(\ln x)\cdot\frac{1}{x} \qquad v = x$

$= uv - \int v\,du$

$= x(\ln x)^2 - \int 2\ln x\, dx$

LET $u = 2\ln x \qquad dv = dx$

$du = \frac{2}{x}\,dx \qquad v = x$

$= x(\ln x)^2 - \left[2(\ln x)x - \int\left(\frac{2}{x}\right)(x)\,dx\right]$

$= x(\ln x)^2 - 2x\ln x + \int 2\,dx$

$= x(\ln x)^2 - 2x\ln x + 2x + C$

(e) *(6 marks)* $\displaystyle\underbrace{\int \frac{5x^3+4x^2-2x-6}{x^4+2x^3+2x^2}\,dx}_{I}$

FACTOR: $x^4+2x^3+2x^2$

$= x^2\underbrace{(x^2+2x+2)}_{\substack{2^2-4(2)(1)<0 \\ \text{IRREDUCIBLE}}}$

$$\frac{5x^3+4x^2-2x-6}{x^2(x^2+2x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+2x+2}$$

$$5x^3+4x^2-2x-6 = Ax(x^2+2x+2) + B(x^2+2x+2) + (Cx+D)x^2$$

$$5x^3+4x^2-2x-6 = Ax^3+2Ax^2+2Ax + Bx^2+2Bx+2B + Cx^3+Dx^2$$

$$= (A+C)x^3 + (2A+B+D)x^2 + (2A+2B)x + 2B$$

$\therefore\ 5 = A+C$	$4 = 2A+B+D$	$-2 = 2A+2B$	$-6 = 2B$
$5 = 2+C$	$4 = 2(2)-3+D$	$\Rightarrow -2 = 2A-6$	$\Rightarrow -3 = B$
$3 = C$	$3 = D$	$4 = 2A$	
		$2 = A$	

$$I = \int \frac{2}{x} - \frac{3}{x^2} + \frac{3x+3}{x^2+2x+2}\,dx$$

$$= \int \frac{2}{x}\,dx - 3\int \frac{1}{x^2}\,dx + 3\int \frac{x+1}{x^2+2x+2}\,dx$$

LET
$u = x^2+2x+2$
$du = (2x+2)\,dx$
$\frac{du}{2(x+1)} = dx$

$$= 2\ln|x| + \frac{3}{x} + \frac{3}{2}\int \frac{\cancel{x+1}}{u}\,\frac{du}{\cancel{x+1}}$$

$$= 2\ln|x| + \frac{3}{x} + \frac{3}{2}\ln|x^2+2x+2| + C$$

Question 6. *(4 marks)* Given that $f(2) = 3$, and that

$$\int_0^2 x f'(x)\, dx = 4$$

find $\int_0^2 f(x) dx$

LET $u = f(x)$ $dv = dx$

$du = f'(x) dx$ $v = x$

$$I = \int_0^2 f(x) dx$$

$$= x f(x) \Big|_0^2 - \int_0^2 x f'(x) dx$$

$$= 2f(2) - (0)f(0) - 4$$

$$= 2(3) - 4$$

$$= 2$$