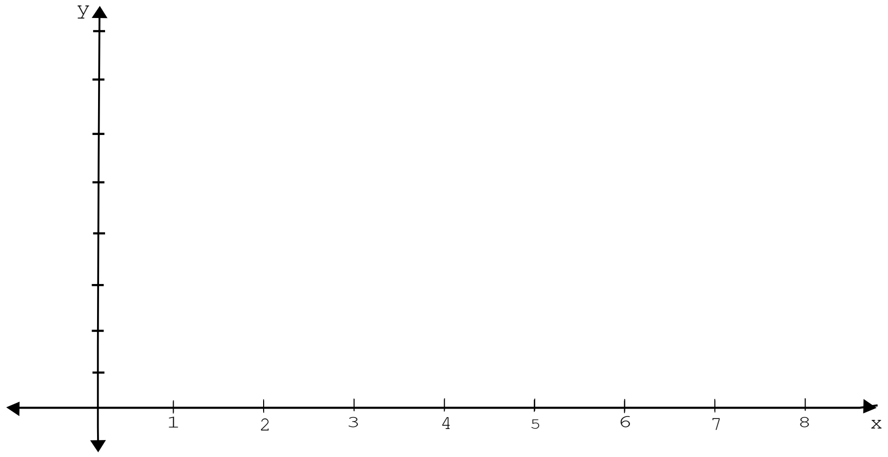
Infinite Sequences Part 2

Let’s graph the sequence$a_n = \frac{1}{n}$ and the function $f(x) = \frac{1}{x}$ on the same graph



It is clear that since

$$\lim_{x\ to \infty} f(x) = 0$$

we must have

$$\lim_{n \to \infty} a_n = 0$$

since the sequence is always "on top of" the function. This idea is expressed in the following theorem:

Theorem: If

$$\lim_{x \to \infty} f(x) = L$$

where L is a finite number or $\pm\infty$ and $f(n) = a_n$ when n is an integer, then

$$\lim_{n \to \infty} a_n = L$$

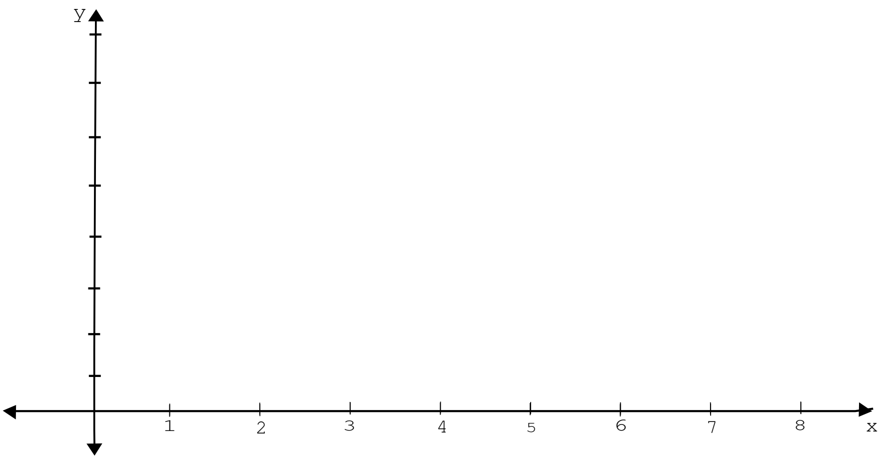
The last example we did satisfies this theorem since

$$f(n) =$$

Be careful! This theorem assumes that L is finite or $\pm\infty$. For example lets look at $a_n = \cos(2n\pi)$ and $f(x) = \cos(2x\pi)$. This satisfies

$$f(n) =$$

But when we look at the graphs



The good news is that this theorem allows us to use L'Hopital's rule for some sequences:

Example: Find the limit of each sequence

1) $a_n = \dfrac{\ln n}{n}$

2) $a_n = \dfrac{\sqrt{n-1}}{4n+5}$

3) $a_n = n^2 e^{-n}$

4) $a_n = \dfrac{2\ln(6n+3)}{4\ln(n+1)}$

5) $a_n = \dfrac{\ln(n+1)}{\sqrt{n}}$

6) $a_n = \dfrac{e^n + n}{\ln n}$

6) $a_n = \dfrac{e^{5n} + n}{e^{2n}}$