EXERCISES Find each limit. Use L'Hôpital's Rule where appropriate. Otherwise use a more elementary method.

1. $\lim_{x\to 3}\dfrac{2x^2-3x-9}{x^2-2x-3}$

2. $\lim_{x\to\infty}\dfrac{4x^3+x-3}{x^2-5x+8}$

3. $\lim_{x\to 1}\dfrac{2x^3-x^2-4x+3}{3x^3-5x^2+x+1}$

4. $\lim_{x\to\infty}\dfrac{6x-5}{4x^2+7x+9}$

5. $\lim_{x\to 0}\dfrac{3x^2+8x}{5x^3}$

6. $\lim_{x\to\infty}\dfrac{x^2-7x-10}{6x^2-x-1}$

7. $\lim_{x\to 0}\dfrac{1-e^x}{2x}$

8. $\lim_{x\to 0}\dfrac{x^2+x}{\sin 3x}$

9. $\lim_{x\to 0^+}\dfrac{\sin x}{1-\cos x}$

10. $\lim_{x\to 0}\dfrac{1-\cos x}{\sin x}$

11. $\lim_{x\to 0}\dfrac{\sin 2x}{\sin x}$

12. $\lim_{x\to 0}\dfrac{e^x-x-1}{5x^2}$

13. $\lim_{x\to\infty}\dfrac{e^{3x}}{\ln x}$

14. $\lim_{x\to 1}\dfrac{x-1}{\sqrt{x}-1}$

15. $\lim_{x\to 0^+}\dfrac{\sqrt{x}}{1-\cos x}$

16. $\lim_{x\to\infty}\dfrac{\sqrt{x-1}}{4x+5}$

17. $\lim_{x\to 1}\dfrac{\ln x}{\sin(x-1)}$

18. $\lim_{x\to\infty}\dfrac{\ln(x+1)}{\sqrt{x}}$

19. $\lim_{x\to\infty}\dfrac{e^x+x}{\ln x}$

20. $\lim_{x\to 1}\dfrac{e^{x-1}-1}{(x-1)^3}$

21. $\lim_{x\to\infty}\dfrac{\ln(x-10)}{\ln(4x+1)}$

22. $\lim_{x\to 0^+}\dfrac{\sqrt{x}}{\ln(x+1)}$

23. $\lim_{x\to\infty}\dfrac{e^{4x}}{e^{3x}+x}$

24. $\lim_{x\to 0}\dfrac{e^{2x}-1}{e^{5x}-1}$

SOLUTIONS

1. $\lim_{x\to 3}\dfrac{2x^2-3x-9}{x^2-2x-3}$ $\left(\text{type } \frac{0}{0}\right)$

$= \lim_{x\to 3}\dfrac{4x-3}{2x-2} = \dfrac{9}{4}$

2. $\lim_{x\to \infty}\dfrac{4x^3+x-3}{x^2-5x+8}$ $\left(\text{type } \frac{\infty}{\infty}\right)$

$= \lim_{x\to \infty}\dfrac{12x^2+1}{2x-5}$ $\left(\text{type } \frac{\infty}{\infty}\right)$

$= \lim_{x\to \infty}\dfrac{24x}{2} = \infty$

3. $\lim_{x\to 1}\dfrac{2x^3-x^2-4x+3}{3x^3-5x^2+x+1}$ $\left(\text{type } \frac{0}{0}\right)$

$= \lim_{x\to 1}\dfrac{6x^2-2x-4}{9x^2-10x+1}$ $\left(\text{type } \frac{0}{0}\right)$

$= \lim_{x\to 1}\dfrac{12x-2}{18x-10} = \dfrac{10}{8} = \dfrac{5}{4}$

4. $\lim_{x\to \infty}\dfrac{6x-5}{4x^2+7x+9}$ $\left(\text{type } \frac{\infty}{\infty}\right)$

$= \lim_{x\to \infty}\dfrac{6}{8x+7} = 0$

5. $\lim_{x\to 0}\dfrac{3x^2+8x}{5x^3}$ $\left(\text{type } \frac{0}{0}\right)$

$= \lim_{x\to 0}\dfrac{6x+8}{15x^2} = \infty$

6. $\lim_{x\to \infty}\dfrac{x^2-7x-10}{6x^2-x-1}$ $\left(\text{type } \frac{\infty}{\infty}\right)$

$= \lim_{x\to \infty}\dfrac{2x-7}{12x-1}$ $\left(\text{type } \frac{\infty}{\infty}\right)$

$= \lim_{x\to \infty}\dfrac{2}{12} = \dfrac{2}{12} = \dfrac{1}{6}$

7. $\lim_{x\to 0}\dfrac{1-e^x}{2x}$ $\left(\text{type } \frac{0}{0}\right)$

$= \lim_{x\to 0}\dfrac{-e^x}{2} = -\dfrac{1}{2}$

8. $\lim_{x\to 0}\dfrac{x^2+x}{\sin 3x}$ $\left(\text{type } \frac{0}{0}\right)$

$= \lim_{x\to 0}\dfrac{2x+1}{3\cos 3x} = \dfrac{1}{3}$

9. $\lim_{x\to 0^+}\dfrac{\sin x}{1-\cos x}$ $\left(\text{type } \frac{0}{0}\right)$

$= \lim_{x\to 0^+}\dfrac{\cos x}{\sin x} = \infty$

10. $\lim_{x\to 0}\dfrac{1-\cos x}{\sin x}$ $\left(\text{type } \frac{0}{0}\right)$

$= \lim_{x\to 0}\dfrac{\sin x}{\cos x} = 0$

11. $\lim_{x\to 0}\dfrac{\sin 2x}{\sin x}$ $\left(\text{type } \frac{0}{0}\right)$

$= \lim_{x\to 0}\dfrac{2\cos 2x}{\cos x} = 2$

12. $\lim_{x\to 0}\dfrac{e^x-x-1}{5x^2}$ $\left(\text{type } \frac{0}{0}\right)$

$= \lim_{x\to 0}\dfrac{e^x-1}{10x}$ $\left(\text{type } \frac{0}{0}\right)$

$= \lim_{x\to 0}\dfrac{e^x}{10} = \dfrac{1}{10}$

13. $\lim_{x\to\infty}\dfrac{e^{3x}}{\ln x}$ $\left(\text{type }\frac{\infty}{\infty}\right)$

$= \lim_{x\to\infty}\dfrac{\left[3e^{3x}\right]}{\left[\frac{1}{x}\right]}$ $\left(\text{type }\frac{\infty}{\infty}\right)$

$= \lim_{x\to\infty} 3x e^{3x} = \infty$

14. $\lim_{x\to 1}\dfrac{x-1}{\sqrt{x}-1}$ $\left(\text{type }\frac{0}{0}\right)$

$= \lim_{x\to 1}\dfrac{[1]}{\left[\frac{1}{2\sqrt{x}}\right]} = 2$

15. $\lim_{x\to 0^+}\dfrac{\sqrt{x}}{1-\cos x}$ $\left(\text{type }\frac{0}{0}\right)$

$= \lim_{x\to 0^+}\dfrac{\left[\frac{1}{2\sqrt{x}}\right]}{[\sin x]}$

$= \lim_{x\to 0^+}\dfrac{1}{2\sqrt{x}\sin x} = \infty$

16. $\lim_{x\to\infty}\dfrac{\sqrt{x-1}}{4x+5}$ $\left(\text{type }\frac{\infty}{\infty}\right)$

$= \lim_{x\to\infty}\dfrac{\left[\frac{1}{2\sqrt{x-1}}\right]}{[4]}$

$= \lim_{x\to\infty}\dfrac{1}{8\sqrt{x-1}} = 0$

17. $\lim_{x\to 1}\dfrac{\ln x}{\sin(x-1)}$ $\left(\text{type }\frac{0}{0}\right)$

$= \lim_{x\to 1}\dfrac{\left[\frac{1}{x}\right]}{[\cos(x-1)]} = 1$

18. $\lim_{x\to\infty}\dfrac{\ln(x+1)}{\sqrt{x}}$ $\left(\text{type }\frac{\infty}{\infty}\right)$

$= \lim_{x\to\infty}\dfrac{\left[\frac{1}{x+1}\right]}{\left[\frac{1}{2\sqrt{x}}\right]}$

$= \lim_{x\to\infty}\dfrac{2\sqrt{x}}{x+1}$ $\left(\text{type }\frac{\infty}{\infty}\right)$

$= \lim_{x\to\infty}\dfrac{\left[\frac{1}{\sqrt{x}}\right]}{[1]} = \lim_{x\to\infty}\dfrac{1}{\sqrt{x}} = 0$

19. $\lim_{x\to\infty}\dfrac{e^x+x}{\ln x}$ $\left(\text{type }\frac{\infty}{\infty}\right)$

$= \lim_{x\to\infty}\dfrac{\left[e^x+1\right]}{\left[\frac{1}{x}\right]}$

$= \lim_{x\to\infty} x\left(e^x+1\right) = \infty$

20. $\lim_{x\to 1}\dfrac{e^{x-1}-1}{(x-1)^3}$ $\left(\text{type }\frac{0}{0}\right)$

$= \lim_{x\to 1}\dfrac{e^{x-1}}{3(x-1)^2} = \infty$

21. $\lim_{x\to\infty}\dfrac{\ln(x-10)}{\ln(4x+1)}$ $\left(\text{type }\frac{\infty}{\infty}\right)$

$$= \lim_{x\to\infty}\frac{\left[\dfrac{1}{x-10}\right]}{\left[\dfrac{4}{4x+1}\right]}$$

$$= \lim_{x\to\infty}\frac{4x+1}{4x-40} \quad \left(\text{type }\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to\infty}\frac{4}{4} = \frac{4}{4} = 1$$

22. $\lim_{x\to 0^+}\dfrac{\sqrt{x}}{\ln(x+1)}$ $\left(\text{type }\frac{0}{0}\right)$

$$= \lim_{x\to 0^+}\frac{\left[\dfrac{1}{2\sqrt{x}}\right]}{\left[\dfrac{1}{x+1}\right]}$$

$$= \lim_{x\to 0^+}\frac{x+1}{2\sqrt{x}} = \infty$$

23. $\lim_{x\to\infty}\dfrac{e^{4x}}{e^{3x}+x}$ $\left(\text{type }\frac{\infty}{\infty}\right)$

$$= \lim_{x\to\infty}\frac{4e^{4x}}{3e^{3x}+1} \quad \left(\text{type }\frac{\infty}{\infty}\right)$$

$$= \lim_{x\to\infty}\frac{16e^{4x}}{9e^{3x}} = \lim_{x\to\infty}\frac{16e^{x}}{9} = \infty$$

24. $\lim_{x\to 0}\dfrac{e^{2x}-1}{e^{5x}-1}$ $\left(\text{type }\frac{0}{0}\right)$

$$= \lim_{x\to 0}\frac{2e^{2x}}{5e^{5x}} = \lim_{x\to 0}\frac{2}{5e^{3x}} = \frac{2}{5}$$