Partial Fractions Part 2

Case 4: The denominator $q(x)$ contains a repeated irreducible quadratic factor ax^2+bx+c, where $b^2-4ac<0$. For each $(ax^2+bx+c)^n$ in the denominator we need

$$\frac{A_1x+B_1}{ax^2+bx+c}+\frac{A_2x+B_2}{(ax^2+bx+c)^2}+\frac{A_3x+B_3}{(ax^2+bx+c)^3}+\ldots+\frac{A_nx+B_n}{(ax^2+bx+c)^n}$$

For example if we wanted to write the following as a sum of simpler fractions we would need

$$\frac{3x^4+2x-1}{x(3x-2)(x^2+x+1)(x^2+1)^2}$$

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Example: Find

1) $\int \frac{1-x+2x^2-x^3}{x(x^2+1)^2}\,dx$

2) $\int \frac{3x^3+9x^2+16x+10}{(x^2+2x+3)^2}\,dx$

Partial fraction examples using mixed cases: Find

1) $\displaystyle\int \frac{2x^3+2x^2-95x+40}{x^3+x^2-20x}\,dx$

2) $\int \frac{x-3}{(x^2+2x+10)^2}\,dx$

3) $\int \frac{3x^4+5x^3+2x+3}{(2x+1)(x^2+1)^2}\,dx$