

Sigma Notation

We often use sigma notation to write sums more compactly. For example:

$$\sum_{i=1}^n f(x_i)\Delta x = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + \dots + f(x_n)\Delta x$$

Here's what it means:

$$\sum_{i=1}^n f(x_i)\Delta x$$

Example:

$$1) \sum_{i=1}^4 i^2 =$$

$$2) \sum_{i=3}^n i =$$

$$3) \sum_{i=1}^7 1 =$$

$$4) 2^3 + 3^3 + 4^3 + \dots + n^3 =$$

Write the following sums in expanded form:

1) $\sum_{i=1}^5 \sqrt{i}$

2) $\sum_{n=1}^6 \frac{1}{n+1}$

3) $\sum_{i=4}^6 (i^3 + 1) =$

4) $\sum_{i=1}^n i^{10}$

Write the following sums in sigma notation:

1) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

2) $\frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \dots + \frac{9}{2}$

3) $\frac{3}{7} + \frac{4}{8} + \frac{5}{9} + \dots + \frac{10}{14}$

4) $2^3 + 2^4 + 2^5 + 2^6 + \dots + 2^n$

Sigma Notation Rules

$$1) \sum_{i=1}^n ca_i =$$

$$2) \sum_{i=1}^n (a_i \pm b_i) =$$

$$3) \sum_{i=1}^n 1$$

$$4) \sum_{i=1}^n i$$

$$5) \sum_{i=1}^n i^2$$

$$6) \sum_{i=1}^n i^3$$

Proof of 4)

Examples:

Evaluate the following

$$\sum_{i=1}^n i(4i^2 - 3)$$

Write in expanded form:

$$1) \sum_{k=5}^8 x^k$$

$$2) \sum_{i=0}^4 \frac{2k-1}{2k+1}$$

$$3) \sum_{j=1}^{n-1} (-1)^j$$

Write in sigma notation:

$$1) 1 + 3 + 5 + 7 + \dots (2n - 1)$$

$$2) 1 - x + x^2 - x^3 + x^4 - x^5$$

$$3) \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36}$$

Find the value of the sum

$$\mathbf{1)} \sum_{i=1}^6 i(i+2)$$

$$\mathbf{2)} \sum_{k=0}^8 \cos k\pi$$

$$\mathbf{3)} \sum_{j=-2}^4 2^{3-i}$$

$$\mathbf{4)} \sum_{k=1}^n 2k$$

$$\mathbf{5)} \sum_{i=1}^n (i^2 + 3i + 4)$$

$$\mathbf{6)} \sum_{i=1}^n i(i+1)(i+2)$$